The role of rotational collapse mode and catenary-type thrust lines in the limit state analysis of masonry arches

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Abstract

Limit state analysis of masonry arches considers the original, hyperstatic structure on the verge of collapse. Minimum thickness analysis as defined by Heyman, based on the principle of thrust line and equilibrium analysis is carried out assuming a certain failure mode. Yet a widely applicable method for determining a-priori the rotational collapse mechanism of a symmetrical arch (to the authors knowledge) is missing. Present paper offers such a method. It employs the geometric relation between the reference line of the arch and the catenary-type thrust line, which latest is the envelope of resultant forces from the external load. Its generalized analytical derivation along with the necessary special stereotomy condition is presented. It is highlighted, that the catenary-type thrust line is the affine image of the moment diagram of the arch – which does not hold generally for thrust lines.

Approximating the thrust line at the limit state to predict the failure mode of an (eminently) circular arch at preliminary stages of FEM/DEM analysis of existing structures is a common practice in the literature [1]. It generally focuses on the effect of various load-cases and the varying locations of the occurring hinges (in general assuming the number of the hinges to be constant). The present study is concerned with the effects of the overall geometry and allows for the prediction of various number of hinges (equal to or larger than 5) – in this regard, it offers an extension to the work of Nikolic [2], who proposed an iterative methodology for pointed circular arches.

The Heymanian assumptions regarding the material behaviour (hence only rotational collapse is considered) accompanied with a simplified structural model is adopted. In specific, the arch is represented by an axis, and the self-load is assigned to the axis regardless of the stereotomy. It is pointed out, that beyond predicting the rotational collapse mode, the catenary type thrust line can be effectively applied for the preliminary analysis of the stability of masonry arches because it results in a bounding value of stereotomy-related minimum thicknesses [3]. However, eventually the rotational collapse mode of the arch selects whether it is a lower or an upper bound – note, that in the former case the estimation is non-conservative. The practical relevance of these results is illustrated via the revisited analysis of the dome of St. Peter's, Rome by Giovanni Poleni: It is pointed out, that instead of the thrust line corresponding to the stereotomy of the dome, Poleni based his assessment on the catenary-type thrust line. By determining the rotational collapse mode of the orange-slice arch by the proposed method of this paper it is concluded, that his results were indeed on the safe side.

References

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