A unified approach for shell analysis on explicitly and implicitly defined surfaces

D. Schöllhammer*[1], T.P. Fries[2]

[1,2] Institute of Structural Analysis, Graz University of Technology, Lessingstr. 25/II, 8010 Graz, Austria, www.ifb.tugraz.at

[1] schoellhammer@tugraz.at, [2] fries@tugraz.at

Abstract

Shells are curved, thin-walled structures and due to their high bearing capacity, they occur in a wide range of applications. For the mechanical modelling, one may distinguish between two classes of shell models based on the fact whether transverse shear deformations are considered or not. These models are well-known as Kirchhoff-Love shells, modelling thin structures, and Reissner-Mindlin shells, suitable for thin and moderately thick shells. Both shell theories result into partial differential equations (PDEs) on surfaces.

In the classical approach of modelling shells, the middle surface is given through a parametrization and based on this map, a local coordinate system with co- and contra-variant base vectors is introduced, which is used for the derivation of the shell equations. Using non-uniform, curvilinear coordinates makes the approach less intuitive and more complex and does not cover implicitly defined shells.

We propose a reformulation of the classical linear shell theory for both models without the explicit introduction of curvilinear coordinates, which applies for parametrized *and* implicit shell descriptions [1, 2]. In particular, the shell equations are recast in the frame of the tangential differential calculus (TDC) [3] using the global Cartesian coordinate system, which simplifies the derivation and the numerical treatment of the obtained PDEs significantly. The reformulation of the shell models including all relevant mechanical quantities and boundary conditions is presented in [1, 2].

For the numerical simulation, one may employ two fundamentally different approaches of solving PDEs on surfaces. The first approach is a classical finite element method, labelled Surface FEM, where the analytical geometry is decomposed into a set of elements, implying element-wise parametrizations. The second approach is labelled as TraceFEM, where the implicitly defined shell is embedded in a 3D background mesh and the shape functions are restricted to the trace of the shell surface. The classical approach is limited to the first approach, whereas the proposed approach is applicable to both situations, which can be seen as a generalization of the classical shell equations.

References

[1] D. Schöllhammer and T.P. Fries, Kirchhoff–Love shell theory based on tangential differential calculus, *Comput. Mech.*, 2018, DOI: 10.1007/s00466-018-1659-5

[2] D. Schöllhammer and T.P. Fries, Reissner–Mindlin shell theory based on tangential differential calculus, *Comp. Methods Appl. Mech. Engrg.*, 2018, **submitted**. Preprint on arXiv: cs.NA 1610.01660

[3] M.C. Delfour and J.P. Zolésio, *Shapes and Geometries: Metrics, Analysis, Differential Calculus, and Optimization*, SIAM, Philadelphia, 2011.