Finding Membrane Shells Subjected to Horizontal Body Forces with Radial Basis Functions

Yu-Chou CHIANG [a]*, Andrew BORGART[a], Qingpeng LI[b]

[a] Faculty of Architecture and Built Environment, Delft University of Technology
* Chiang.YuChou@gmail.com

[b] School of Architecture and Urban Planning, Nanjing University

Abstract

Membrane shells, which have minimized bending movements, are regarded as ideal structural forms in terms of material efficiency. Most of numerical form-finding methods are based on discretizing membranes into funicular networks and focusing on gravitational loading only.

In order to discover smooth solutions of shells and incorporating horizontal loads, this paper present a method to find the equilibrium forms of the membrane shells by solving Pucher's equation [1]:

$$\left(\frac{\partial^2 F}{\partial x_2^2} - \int \bar{p}_1 dx_1\right)\frac{\partial^2 z}{\partial x_1^2} + 2\left(-\frac{\partial^2 F}{\partial x_1 \partial x_2}\right)\frac{\partial^2 z}{\partial x_1 \partial x_2} + \left(\frac{\partial^2 F}{\partial x_1^2} - \int \bar{p}_2 dx_2\right)\frac{\partial^2 z}{\partial x_2^2} - \bar{p}_1\frac{\partial z}{\partial x_1} - \bar{p}_2\frac{\partial z}{\partial x_2} = -\bar{p}_z,$$

where $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_z$ are the horizontal and vertical load components; $F(\mathbf{x})$ is the stress function and $z(\mathbf{x})$ is the shape of the shell. We utilize radial base functions (RBFs) to represent stresses and shapes of the membranes [2]:

$$\tilde{F}(\mathbf{x}) = \sum_{i=1}^{n} w_{F_i}\phi_F(\|\mathbf{x}-\boldsymbol{\mu}_{F_i}\|;\rho_{F_i}) + h_F(\mathbf{x}), \text{ and } \tilde{z}(\mathbf{x}) = \sum_{i=1}^{n} w_{z_i}\phi_z(\|\mathbf{x}-\boldsymbol{\mu}_{z_i}\|;\rho_{z_i}) + h_z(\mathbf{x}),$$

where $\phi_i(\mathbf{x})$ are the radial functions, $\mathbf{x}$ is the evaluation point, w_i the weights, $\boldsymbol{\mu}_i$ source points, ρ_i scale parameters, $h(\mathbf{x})$ polynomial terms. This paper applies least square method (LSM) to find the weights and the polynomials which fit the boundary conditions (i.e. zero stresses at the free edges) and the governing equation (i.e. Pucher's equation). When all the parameters are carefully chosen, the stress function $\tilde{F}(\mathbf{x})$ and shape function $\tilde{z}(\mathbf{x})$ can deliver sufficient accuracy. The presented method has been preliminarily implemented to find a shell on a triangle-shaped ground plan incorporating horizontal loads. The form-found geometry is then analyzed with a finite element model[3]. The result confirms that the form-found shell has the stress distribution similar to the prescribed stresses.

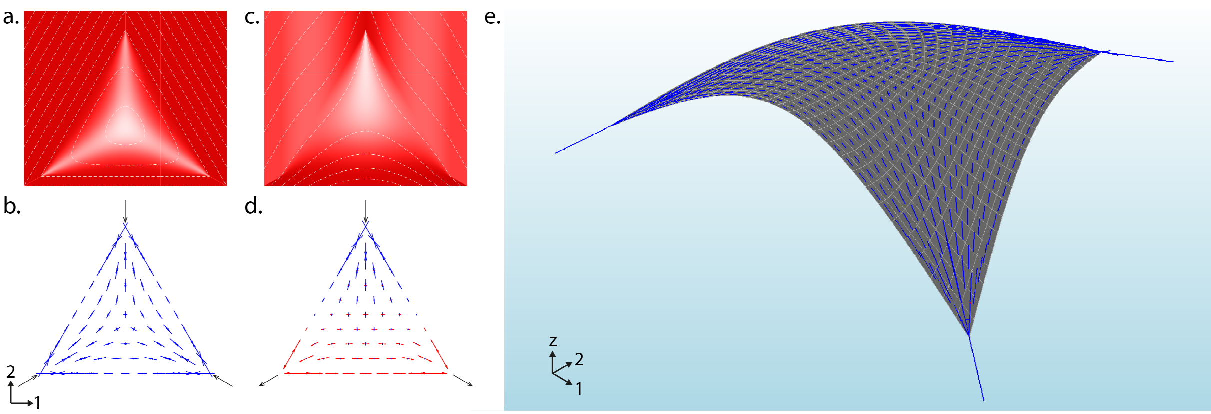

Figure 1. Preliminary results: A stress function when symmetrical compression force applied at the point supports and its stress field (a & b), another stress function when the shell subjects to horizontal load in the direction of axis-2 and its stress field (c & d), and the principle stresses from FEM analysis on the form-found membrane, in which 30% self-weight horizontal load is included (e).

References

[1] P. Csonka, *Theory and practice of membrane shells*. VDI Verlag, 1987.
[2] M. D. Buhamann, *Radial Basis Functions*. Cambridge, 2003.
[3] DIANA FEA BV, *DIANA*. Delft, 2018. [Online]. Availiable: https://dianafea.com.