Form-finding method of cable-net structures using Grassmann algebra and mass-points

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Abstract

In this paper we will describe a form-finding method for cable-net and self-supported structures using Grassmann algebra and weighted points called mass-points. In Grassmann algebra points are expressed in the form x=(m, x_1, x_2, x_3), where $m \neq 0$ is a (generalized) mass of the point, and $\mathbf{x}$=$(x_1/m, x_2/m, x_3/m)$ represents position of the point in Cartesian coordinates, and also a position vector of the point x in Grassmann algebra. By adding masses to points it is possible to study geometry applying basic principles of mechanics [1]. In classical mechanics, points represent locations in space, and vector represent forces acting on that points with added masses. Using Grassmann algebra and algebra of weighted points, centre of given mass-points can be found using only its masses and coordinates. Summation of the coordinates of the given mass-points gives us the centre of the mass-points system. We can use expression: $(m_c, m_c\mathbf{x}_c) = (\sum_{n=1}^{n} m_i, \sum_{n=1}^{n} m_i\mathbf{x}_i)$ [2]. This will be described in the first part of the paper and shown as an example of a simple cable-net system with one free node. The procedure in form diagram will be simultaneously followed by reciprocal polygon of internal forces in force diagram.

Finding centre of mass-points of a free node is closely related to form-finding of cable-net and self-supported structures using well-known Force Density Method (FDM). The method presented in this paper is iterative, based on mass-points, where points represent fixed nodes, masses represent force densities in element of structure, and centre of the mass-points represents position of the free node, whose coordinates are unknown. This will be shown in an example of a cable-net structure. The method is similar to the one explained in [3].

In the last part of the paper we will show construction of the reciprocal force polygon using obtained centres of the given mass-points. It should be also noted that the idea of using mass-points and basic principles of mechanics leads to the method of the Dynamic Relaxation Method.

References

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[3] L. Enrique, P. Block. and J. Schwartz, "Form-finding method for prestress-dominant surface structures using graphic statics", *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2015*, August, 2015, Amsterdam, Netherlands.