Application of Graphic Statics and Theory of Plasticity to the design of strut-and-tie models in reinforced concrete

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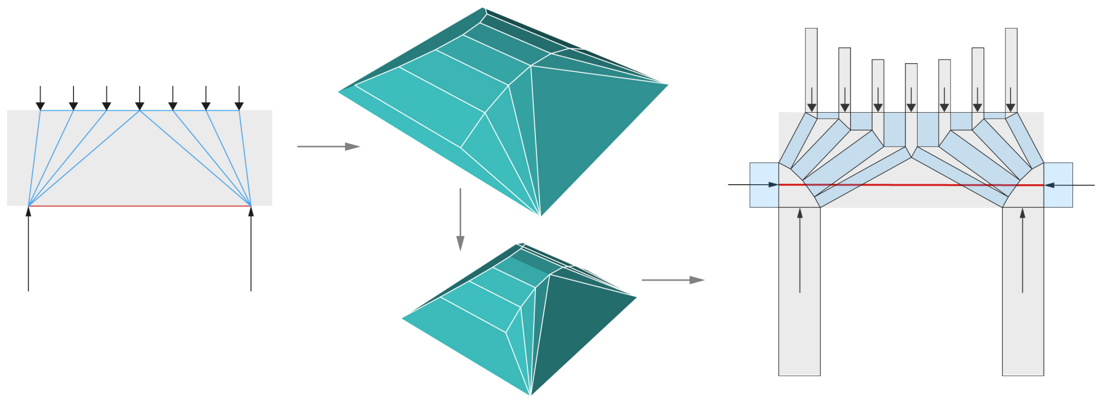
Abstract

Graphic statics are currently experiencing a renaissance for the analysis and design of structures in static equilibrium while also showing the potential of being applied to the solution of a wide range of problems in other fields of structural engineering. The constitutive objects of this geometrical methodology are the 'form' and 'force' diagrams, which are interlinked through the notion of reciprocity. Grounded on the intellectual legacy of J. C. Maxwell, it has been recently shown [1] that graphic statics can provide a unified, direct, and purely geometrical framework for the static analysis of planar and spatial trusses that are projections of polyhedral stress functions. These structures can be self-stressed or subjected to external loads and need not be compression-only or tension-only. Within this context, a useful generalisation is the definition of 'Minkowski sum' [3][4], which combines form, force diagrams, and uniform hydrostatic stresses in a single diagram.

Strut-and-tie models based on the theory of plasticity can be used as powerful tools for the analysis and design of force flows and stress fields in reinforced concrete structures [2]. At present, however, no general approach exists for their automatic generation. This paper aims to introduce a geometrical methodology to generate stress fields via graphic statics, by interlinking form, force diagrams, their stress functions, and Minkowski sums. Based on this proposed approach, for a given form diagram, its polyhedral Airy stress function is transformed through a series of geometrical manipulations to produce a valid stress field that respects specified topology of the strut-and-tie model, boundary constraints, and applied external forces.



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