Frame model for analysis and form generation of rigid origami for deployable roof structure

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Abstract

There are various methods for design and analysis of rigid origami whose facets do not deform throughout the process of folding. However, very few methods have been proposed for form generation of rigid origami with a simple but not regular crease pattern. It is important to obtain a simple crease pattern in terms of constructability when we apply rigid origami to deployable structures in architecture. In this study, we present a method of obtaining a developable and rigid-foldable polyhedron that approximates a curved surface with a simple crease pattern using a frame model.

In general, rigid origami is analyzed using the unstable truss model or the rotational hinges model [1]. However, the former model needs many members to model a quadrilateral or more polygonal facet. The latter model is not suitable to carry out large-deformation analysis using a general finite element analysis software. Therefore, we have developed a frame model which enables us to model a polyhedral origami with few elements and to use the same variables in form generation, evaluation of kinematic indeterminacy [2], and large-deformation analysis. In a frame model, frame elements connecting the node on a crease line or an outer edge and the node in a facet are used to model a polyhedral origami. Frame elements are connected by hinges on crease lines and rigidly connected in facets.

In form generation, we use the optimization method to obtain a polyhedron which satisfies the conditions for making it developable and reducing its degree of freedom (DOF) of deformation. The variables are coordinates of nodes on crease lines and outer edges, and we can arbitrarily define coordinates of nodes in facets. When a polyhedral origami is developable to a plane, the sum of angles between adjacent crease lines around each interior vertex needs to be equal to 2π. Thus, we formulate an optimization problem to minimize the sum of errors of angles at all interior vertices so that a developable and rigid-foldable origami is generated. Optimization starts from a triangulated target surface to be approximated by a polyhedral origami. To reduce the DOF, constraints are sequentially assigned so that the specified pair of normal vectors of adjacent triangular facets are parallel. By removing (fixing) the crease lines between them, we can obtain a polyhedron which has both triangular and quadrilateral flat facets. To confirm the existence of infinitesimal folding mechanism, singular value decomposition is carried out in the same manner as the frame model of linkage mechanism [2]. In addition, it is verified by large-deformation analysis that the polyhedron can be continuously developed to a plane without deformation of each facet.

It is confirmed in the numerical examples that a developable and rigid-foldable polyhedral origami with various shapes can be generated using the proposed process of optimization. We can easily reduce the DOF of mechanism by assigning constraint to remove the crease lines to generate quadrilateral faces.

References

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