Preservation of Local Bounds During Transport of High-Order Finite Element Fields

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ABSTRACT

In this presentation we consider a discretization of the transport equation with high-order discontinuous finite elements. The main motivation behind this work is the need to derive a method applicable to finite element spaces above Q_3. As demonstrated in [2], classical algebraic Flux Corrected Transport (FCT) techniques, for example [1], are well suited only for polynomial orders up to three. For finite elements above Q_3, one observes severe local oscillations that occur within the admissible bounds. The primary reason is that as the polynomial order increases, the neighborhood used for defining admissible bounds grows, allowing higher variation.

We address this issue by describing a new algebraic FCT-like method for the standard transport equation. We briefly review the underlying Discontinuous Galerkin (DG) finite element discretization, the properties of utilized Bernstein polynomial spaces, low-order admissible solution, and classical FCT. Next we introduce the concept of localized stencils, which use topological information to define tighter bounds for each degree of freedom. This is combined with an element-based FCT formulation where instead of adjusting all incoming fluxes, one adjusts the value at each degree of freedom, causing a mass error in each cell. These mass errors are corrected by a sub-zonal mass redistribution process that results in a simple non-linear problem to be solved in each cell. We show that the new method reduces undesired local oscillations, while still achieving the convergence rates of the classical FCT method reported in [1] for smooth profiles, i.e., order $p+1$ in monotone regions and rates of at most $2-2.5$ around local extrema.

The new capabilities allow us to use higher order finite element spaces in our target application, which is the remap phase of Arbitrary Lagrangian-Eulerian (ALE) hydrodynamics. We demonstrate that the method is applicable to this framework and present 2D and 3D results in the context of high-order finite element advection remap.

REFERENCES

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