A multidimensional stabilization technique for coupled convection-diffusion-reaction equations

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A large variety of processes of practical interest in several branches of science such as biology, chemistry, economics, physics, physiology and social science, give rise to partial differential equations having diffusive, convective and reactive terms. Moreover, in some instances several species or components interact with each other rendering strongly coupled systems of convection-diffusion-reaction equations. When classical numerical methods are used to approximate the solutions of such equations, even in the simplest one dimensional case in the steady state regime for a single equation, instabilities are found when the convective or reactive terms dominate over the diffusive term.

Stabilization techniques have been developed to handle such transport equations by numerical means. Nevertheless, no extensive work has been carried out for systems of coupled equations [1]. The reason for this immaturity is the lack of a maximum principle when going from a single transport equation towards systems of coupled equations [3].

The main aim of this communication is to present a stabilization technique for systems of coupled convection-diffusion-reaction equations in two or three dimensions based on coefficient perturbations. These perturbations are optimally chosen in such a way that certain compatibility conditions analogous to a maximum principle are satisfied. Once the computed perturbations are injected in the classical Bubnov-Galerkin finite element method, they render smooth and stable numerical approximations. This methodology extends the recently proposed perturbation-based approach in one dimension and for specific equations [2] to a general system of convection-diffusion-reaction equations in multiple space dimensions.

Applications to several prototypical two-dimensional coupled systems of partial differential equations are presented. These results allow envisioning the use of the developed technique for simulating systems of strongly coupled convection-diffusion-reaction equations with an affordable computational effort. An analysis of the stabilization technique using Toeplitz operator theory is briefly discussed as well.

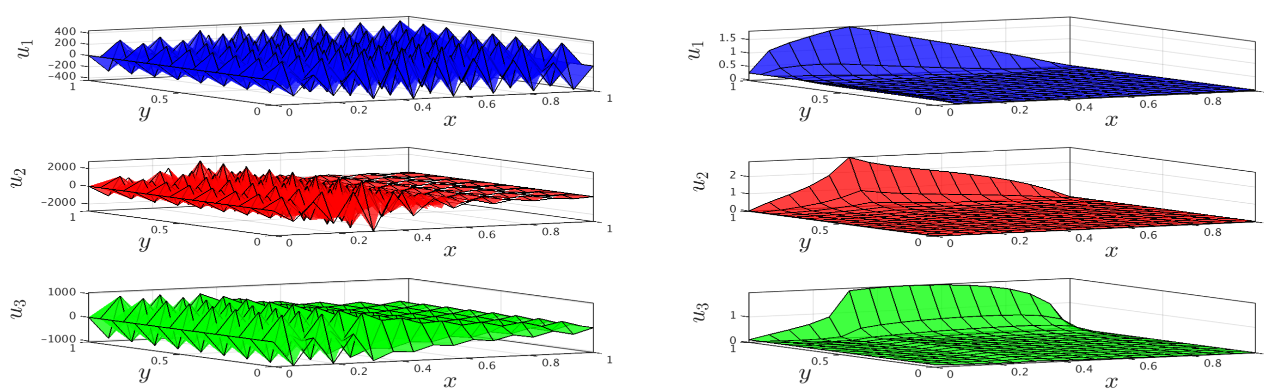

Figure 1: Comparison of the numerical approximations obtained for a coupled convection-diffusion-reaction system with the classical (left) and stabilized (right) scheme. Note the oscillations in the approximation obtained with the classical scheme while the stabilized scheme yields smooth and stable approximations.

References

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