Preconditioning elliptic operators in high-performance all-scale atmospheric models on unstructured meshes

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ABSTRACT

Effective simulation of all-scale atmospheric flows – e.g., cloud-resolving global weather – involves semi-implicit integration of the non-hydrostatic compressible Euler equations under gravity on a rotating sphere. Such integrations depend on complex non-symmetric elliptic solvers. The condition number of the underlying sparse linear operator is $O(10^{10})$, which necessitates bespoke operator preconditioning. This paper highlights the development of a specialised deflation/multigrid preconditioner for the non-symmetric Krylov-subspace solver. This development is set in the context of a massively-parallel high-performance computing environment at ECMWF, aimed at architectures evolving towards exascale [1].

The employed numerical models are based on non-oscillatory forward-in-time integrators that rely on the Multidimensional Positive Definite Advection Transport Algorithm (MPDATA) and a robust non-symmetric Krylov solver, formulated on arbitrary hybrid computational meshes discretising the generalised curvilinear framework of the computational space [2]. In the Krylov solver, the deflation preconditioner [3] primarily removes the inherent stiffness due to the anisotropy of the terrestrial atmosphere – geometrically a thin shell. The multigrid machinery further relaxes the horizontal stiffness due to the high-resolution discretisation and variability of the metric coefficients with the Jacobian vanishing at the poles. Theoretical considerations are illustrated with simulations of stratified turbulent flows and baroclinic instability [4].

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