Application of Hybridizable discontinuous Galerkin to fluid flow/heat transfer problems and coupling with finite elements

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ABSTRACT

Hybridizable discontinuous Galerkin (HDG) [1] methods are introduced to address some of the shortcomings of Discontinuous Galerkin (DG) methods. Ever since, HDG is applied to variety of problems in both fluid and solid mechanics. On the one hand, the present work aims at comparing computational efficiency between HDG and continuous Galerkin (CG) methods in fluid flow problems. On the other hand, formulations are derived for coupling of incompressible Navier–Stokes equations with convection-diffusion equation using HDG and for coupling of HDG with CG method for Laplace equation.

Firstly, a comparison of computational efficiency between HDG and CG is presented for Stokes and Navier–Stokes problems [2]. Higher-order meshes (up to ninth degree) of triangular and quadrilateral elements are used in the comparison study. It is followed by a study on stability between HDG and CG using a manufactured solution that has a sharp boundary layer. Then after, coupling between Navier–Stokes equations and convection-diffusion is proposed for HDG and convergence results are presented for benchmark problems. Results of natural buoyancy flow for different Rayleigh numbers are presented for HDG and CG methods. Finally, a coupling strategy between HDG and CG for Laplace's equation is presented.

It is observed that computational efficiency of HDG is similar, and sometimes superior, to CG for a given level of accuracy in the case of benchmark problems. Numerical results suggests that HDG has better stability properties than its CG counterpart and this fact is demonstrated by comparing both solutions, where numerical instabilities can be noticed in the case of CG. The coupling scheme between HDG and CG is tested with a benchmark example and convergence plots for variables of interest are presented. This scheme is then extended to solve the conjugate heat transfer problem between a solid and a fluid domain.

REFERENCES

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