Iterative solvers based on the discrete gradient method for symmetric positive definite linear systems

Y. Miyatake*, T. Sogabe* and S.-L. Zhang*

* Department of Computational Science and Engineering, Graduate School of Engineering, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8603, Japan
e-mail: miyatake@na.nuap.nagoya-u.ac.jp,
sogabe@na.nuap.nagoya-u.ac.jp,
zhang@na.nuap.nagoya-u.ac.jp

ABSTRACT

In this talk, we propose a new framework for generating iterative solvers for symmetric positive definite linear systems.

It is known that the exact solution of such linear systems can be interpreted as the equilibrium of a certain gradient system. One could thus expect that applying ODE solvers, such as Euler and Runge-Kutta methods, to the gradient system gives linear solvers. However, standard ODE solvers tend to require an extremely small stepsize to guarantee the convergence, and thus the resulting linear solvers are in most cases computationally less efficient than existing ones.

In this talk, to lift the restriction on the stepsize, we focus on the so-called discrete gradient method [1, 2], which has been developed in the context of numerical ordinary differential equations in the last two decades. The main characteristic of the method is that, when it is applied to gradient systems, the numerical solution always converges to the equilibrium, independently of the choice of stepsize. In this talk, we show that applying the discrete gradient method to appropriate gradient systems generate a number of linear solvers, which include several well-known stationary iterative solvers such as the SOR method, and the computational efficiency of the derived linear solvers can be improved by devising the stepsize.

REFERENCES

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