Spectral Investigations of Nitsche's Method

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ABSTRACT

Incompatible discretization methods provide added flexibility in computation by allowing meshes to be unaligned with geometric features and easily accommodating non-interpolatory approximations. Such formulations that are based on Nitsche's approach to enforce surface constraints weakly, which shares features with stabilized methods, combine conceptual simplicity and computational efficiency with robust performance. The basic workings of the method are well understood, in terms of a bound on the parameter. However, its spectral behavior has not been explored in depth. Such investigations can shed light on properties of the operator that effect the solution of boundary-value problems. Furthermore, incompatible discretizations are rarely used for eigenvalue problems. Our spectral investigations lead to practical procedures for solving eigenvalue problems that are formulated by Nitsche's approach.

We consider an eigenvalue problem for the Laplacian with Dirichlet boundary conditions enforced by Nitsche's approach. The standard discrete formulation gives rise to a finite number of approximate eigenpairs, referred to as 'physical' since they approximate the lower exact eigenpairs. On the same mesh, Nitsche's formulation has additional degrees-of-freedom (on the boundary), and therefore additional eigenpairs. These are associated with the constraints, and are not approximations of exact eigenpairs. We show that the sensitivity of the eigenvalues to the Nitsche parameter is equal to a boundary quotient of the eigenfunctions, akin to the Rayleigh quotient, but with a boundary norm replacing the energy norm in the numerator.

These features are explored by numerical studies in a rectangular domain. Starting without stabilization, the physical and constraint eigenpairs are identified clearly by the behavior of the eigenfunctions on the boundary, quantified by the boundary quotient. Overall, increasing the stabilization parameter raises the constraint eigenvalues roughly linearly, but has little effect on the physical eigenvalues. This is consistent with the sensitivity result in terms of the boundary quotient. In the practically important mid-range of stabilization, intricate behavior of eigenvalue veering along with eigenfunction coupling is exhibited.

Algebraic elimination of the Nitsche boundary degrees-of-freedom results in a reduced system with a spectrum that closely approximates the physical eigenpairs and is fairly insensitive to stabilization (provided the usual bound on the parameter is kept). This indicates that the peculiar behavior associated with eigenvalue veering is innocuous. This conclusion is supported by numerical tests of boundary-value problems.

For eigenvalue problems formed by Nitsche's approach (again, with the usual bound on the parameter), using Irons-Guyan reduction results in a system that retains the symmetric banded structure of the original formulation, and is free of the constraint eigenpairs, thereby amenable to solution by standard eigenvalue solvers.

These studies are extended to the parameter-free, non-symmetric variant of Nitsche's method.