A projection based variational multiscale method for ALE form of incompressible Navier-Stokes equations in moving domains

Birupaksha Pal*, **Sashikumaar Ganesan**

Computational Mathematics Group
Department of Computational and Data Sciences
Indian Institute of Science, Bangalore, India
e-mail: birupaksha.pal@gmail.com, sashi@cds.iisc.ac.in

ABSTRACT

Turbulent flows are highly unsteady flows, characterized by chaotic behavior, where the main velocity flow field is superimposed by random velocity fluctuations. Two popular approaches for the numerical simulation of turbulent flows are the direct numerical simulation (DNS) and the large eddy simulation (LES). DNS is a brute force approach that involves solving the Navier-Stokes equations numerically by resolving all scales by an appropriate fine mesh, whereas in LES the scale separation is handled by some filter function and the larger scales of the flow are resolved by the mesh with the effect of small unresolved scales on the large resolved scales being incorporated by a turbulence model. Variational Multiscale method (VMS) [1,2,3] is a significant new improvement on the classical LES approach, which do away with the commutation errors arising from filtering. VMS also allows for separation of the entire range of scales in the flow field into two or three groups. Thus enabling a different numerical treatment for different scales. In addition to the existing challenges, the computational complexity of the problem increases further when a time-dependent domain is considered.

In this presentation an arbitrary Lagrangian-Eulerian (ALE) based variational multiscale method for computations of incompressible Navier-Stokes equations in deforming domains will be presented. The numerical scheme, based on [4], is a three-scale VMS with a projection based scale separation, where the large scales are represented by an additional tensor valued space. The resolved large and small scales are computed in a coupled way with the effects of unresolved scales confined only to the resolved small scales. The popular Smagorinsky eddy viscosity model is used to approximate the effects of unresolved scales. The used ALE approach consists of an elastic mesh update technique. Moreover, a computationally efficient scheme is obtained by the choice of orthogonal finite element basis function for the resolved large scales. Simulations of flow over an oscillating beam and over a plunging aerofoil will be presented to validate the proposed scheme.

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