A High-Order Parallel Implicit Newton-Krylov-Schur Algorithm for Steady and Unsteady Flows Based On Summation-By-Parts Operators

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ABSTRACT

This presentation will describe an efficient and robust high-order parallel implicit algorithm for the numerical solution of the compressible Navier-Stokes and Reynolds-averaged Navier-Stokes equations applicable to steady fluid flows and time-accurate computations of unsteady flows [1]. High-order accuracy in space is obtained through the use of summation-by-parts operators with provable energy stability. High-order accuracy in time is obtained through various diagonallyimplicit methods of Runge-Kutta type, including multistep variants. Nonlinear system solution for both steady and unsteady flows is achieved through the Jacobian-free Newton-Krylov approach combining an inexact-Newton method with a preconditioned Krylov solver for nonsymmetric linear systems. All equations, including the one-equation turbulence model, are fully coupled and analytically differentiated. Excellent parallel scaling is achieved through the use of an approximate Schur preconditioner. For steady flows, two options are available to determine a suitable initial iterate for the Newton method, the classical pseudo-transient continuation approach and a recently developed monolithic homotopy continuation algorithm [2]. The presentation will describe opportunities arising from a recent generalization of the summation-by-parts concept [3] and the close relationship to the discontinuous Galerkin finite element method [4]. After providing an overview of the underlying algorithms, the presentation will provide several examples demonstrating the accuracy, efficiency, robustness, and parallel scaling of the algorithm. These will include Reynolds-averaged Navier-Stokes solutions for flows over the ONERA M6 wing as well as the wing-fuselage geometry used for the 5th Drag Prediction Workshop as well as large eddy simulations of the flow over the SD7003 airfoil and the Taylor-Green vortex flow. Results shown will include comparisons with experiment and other computations as well as characterization of efficiency and parallel scaling.

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