Generalized Ambrosio-Tortorelli Models for Crack Propagation in Brittle Materials

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ABSTRACT

A very popular model of quasi-static brittle fracture is the Ambrosio-Tortorelli (AT) functional. In the AT framework, the crack is identified by a smooth phase-field, and the minimization of the functional is obtained by solving elliptic boundary value problems, numerically tackled via a finite element method. When the ratio between domain size and crack thickness is large, the computational cost can become extremely high, due to the need of fully capturing both scales. An efficient approach to face this issue can be based on mesh adaptation, driven by an a posteriori error estimator, which allows one to refine the grid only in a thin neighborhood of the crack (see, e.g., [5]). In [1, 2, 3], an anisotropic error estimator and a new minimization algorithm are proposed and applied to the classical AT approximation, in the case of both anti-plane and plane-strain isotropic linear elasticity. Several numerical tests assess the reliability of the whole adaptation procedure. The employment of an anisotropic grid allows one to considerably reduce the number of mesh elements in comparison with other (isotropic) adaptation techniques. In this communication, we extend the approach presented in [1, 2, 3] to the generalized AT model considered in [5], where the AT functional is modified in order to deal with several types of constitutive laws. Moreover, we address the extension proposed in [4] for including thermal inelastic effects which can model both genesis and propagation of cracks in brittle materials. The numerical verification performed on standard benchmarks confirms the good accuracy guaranteed by anisotropic grids, as well as a significant computational saving.

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