Conservative DEC Discretization of Incompressible Navier-Stokes Equations on Arbitrary Surface Simplicial Meshes With Applications

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A conservative discretization of incompressible Navier-Stokes equations over surface simplicial meshes is developed using discrete exterior calculus (DEC). The DEC discretization is carried out on the exterior calculus form of Navier-Stokes equations, where the velocity field is represented by a 1-form. A distinguishing feature of our method is the use of an algebraic discretization of the interior product operator and a combinatorial discretization of the wedge product. Numerical experiments for flows over surfaces reveal a second order accuracy for the developed scheme when using structured-triangular meshes, and first order accuracy for otherwise unstructured meshes. The mimetic character of many of the DEC operators provides exact conservation of both mass and vorticity, in addition to superior kinetic energy conservation. The employment of various discrete Hodge star definitions based on both circumcentric and barycentric dual meshes is also demonstrated. The barycentric Hodge star allows the discretization to admit arbitrary simplicial meshes instead of being limited only to Delaunay meshes, as in previous DEC-based discretizations. The convergence order attained through the circumcentric Hodge operator is retained when using the barycentric Hodge, with additional super-convergence behavior when using unstructured simplicial meshes generated by successive subdivision of coarser meshes. The discretization scheme is presented in detail along with numerical test cases demonstrating its numerical convergence and conservation properties. More advanced test cases exploring the physics of flows over curved surfaces are also demonstrated.