# DG based sharp interface methods for multi-phase problems: numerical challenges, validation difficulties and a forecast to new areas of application

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## ABSTRACT

In the last decade, group sharp interphase methods have been developed to overcome essential difficulties of smooth interface methods and to adequately deal with non-smooth problems such as multi-phase problems. They have the inherent difficulty that various physical quantities are non-smooth across the interphase. These are in particular the material properties viscosity and density, which, in conjunction with surface tension, lead to singularities i.e. kinks in the velocity and jumps in the pressure. At the same time, the interphase moves arbitrarily across cell-boundaries. These peculiarities require a very special treatment, in particular if high order methods such Discontinuous Galerkin (DG) are employed, such that its convergence order is preserved.

A broad variety of methods had to be developed to cope with problems such as: (i) fast and high-order integration of cut-cells, (ii) surface tension, (iii) small cells leading to high condition numbers, (iv) pre-conditioning because of high density and viscosity ratios (v) high-order time integration (vi) accurate, mass-preserving level-set algorithms to name only a few.

Though rather sophisticated high order methods have been developed to adequately deal with the latter problems, they still lack maturity e.g. long-term stability, if integrated into a single solver.

Further, testing of the code against analytic solutions of the Navier-Stokes equations is particularly challenging, as essentially all existing solutions of Navier-Stokes have been developed only for single-phase flows, or, in other words, solution for multi-phase problems are essentially non-existing.

Here a new method, called Fokas unifying method, seems to overcome the mayor mathematical obstacles, as it either allows to explicitly construct analytic solutions for multi-phase problems or at least reduces them to a simpler problem. Examples will be presented both for problems where the interface is following a prescribed path or freely moves according to a free boundary value problem.

Beside multi-phase flows there are other continuum problems with sharp interphases such as shocks or certain combustion models. An outlook will be given which problems may be tackled with sharp interphase methods.

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