# Isogeometric Analysis of the Navier–Stokes–Cahn–Hilliard equations with application to incompressible two-phase flows

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## ABSTRACT

In this work, we present our numerical results of the application of Galerkin-based Isogeometric Analysis (IGA) to incompressible Navier–Stokes–Cahn–Hilliard (NSCH) equations in velocity–pressure–phase field–chemical potential formulation. For the approximation of the velocity and pressure fields, LBB compatible non-uniform rational B-spline spaces are used which can be regarded as smooth generalizations of Taylor–Hood pairs of finite element spaces. The one-step $\theta$-scheme is used for the discretization in time. The static and rising bubble, in addition to the nonlinear Rayleigh-Taylor instability flow problems, are considered in two dimensions as model problems in order to investigate the numerical properties of the scheme. Our results obtained from the application of Isogeometric Analysis to a phase field-based two-phase flow model are shown to be in very good consensus with those obtained from sharp interface models. We therefore conclude that the above combination, i.e. Isogeometric Analysis and phase field-based two-phase flow models constitute a very good choice in the treatment of two-phase flow problems, in particular in combination with complex geometries.

*Keywords*: Two-phase flow, Cahn–Hilliard phase field model, Navier–Stokes–Cahn–Hilliard equations, Isogeometric Analysis, Isogeometric finite elements, B-splines/NURBS , Rising bubble, Rayleigh–Taylor instability

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