High-order, Smooth and Pointwise Divergence-free Vector Field Discretization of the Incompressible Navier-Stokes Equations on Simplicial Meshes

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ABSTRACT

We introduce high-order smooth spaces of discrete differential forms on simplicial meshes. The construction is based on convolution of classical Whitney forms [1] with the Courant element and relies on the property that convolution commutes with the exterior derivative. The resulting spaces follow a De Rham sequence and form a basis for the finite element exterior calculus [2].

We apply the compatible spaces of discrete differential forms to the incompressible Navier-Stokes equations. The resulting discretization features a point-wise divergence free velocity field and automatically satisfies the inf-sup stability criteria. Several numerical test cases illustrate the efficacy of the proposed methodology.

REFERENCES

[1] D. Arnold, R. Falk, R. Winther, "Finite element exterior calculus, homological techniques, and applications", *Acta Numerica*., Vol. **15**, (2006).

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