# Numerical Study of Discontinuous Galerkin Schemes for Higher Order Moment Equations Arising from Rarefied Gas Dynamics

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## ABSTRACT

The Boltzmann's equation accurately describes the flow of a gas for all ranges of the Knudsen number. In the present work we focus upon the solution of the Boltzmann's equation, for boundary value problems, with the help of moment equations which are based upon the Grad's distribution function [1]. A methodology to construct boundary conditions for these moment equations was already proposed in [1] where the Maxwell's accommodation model was used. As has been discussed in [2], to properly capture the boundary layers of non-equilibrium flows higher order moment equations are needed; [2] discusses the convergence of large moment systems for boundary value problems. It has been shown previously that Maxwell's boundary conditions can provide us with inaccurate results for certain flow regimes therefore we will firstly derive a set of boundary conditions which leads to well-posedness.

The accuracy of the higher order moment systems motivates us to develop a computational framework for flow problems involving complex geometries; the present work is an initial step towards this end. In the present work we will present a Discontinuous Galerkin(DG) discreetization of the moment equations. It has been shown in the previous works that a weak enforcement of the boundary conditions for well-posed systems can provide us with a stable discreetization if the numerical scheme can be written in the Summation-By-Parts(SBPs) form. Using the fact that DG schemes fall into the category of SBP schems, we will present a weak discreetization of the boundary conditions using characteristic splitting; the stability of the discreetization will follow from energy estimates.

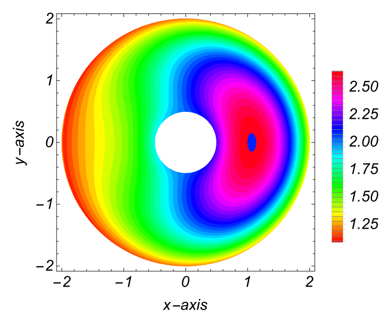


**Figure 1**: Variation of $\theta$(temperature in energy units) for $Kn = 0.1$ corresponding to a moment sub-system derived from Grad's-26 moment equations. .

The numerical framework will then be used to study some benchmark problems involving curved boundaries; Fig-1 shows the variation of temperature(in energy units) for a heat conduction problem involving two concentric cylinders corresponding to a moment sub-system derived from Grad's-26 moment equations. As expected, the discreetization proves to be stable near the boundary. In addition to the characteristic splitting at the boundary, the moment equations also provide us with a stable boundary discreetization if one expresses the odd moments in terms of the even ones; we will compare both the implementations in terms of the approximation quality.

## REFERENCES

[1] Grad, H., On the kinetic theory of rarefied gases. *Communications on Pure and Applied Mathematics*, pp. 331–407, (1949)

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