Residual-Based Large Eddy Simulation Turbulence Models for Incompressible Magnetohydrodyanmics

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ABSTRACT

The goal of this work was to develop, introduce, and test a promising computational paradigm for the development of turbulence models for incompressible magnetohydrodynamics (MHD). MHD governs the behavior of an electrically conducting fluid in the presence of an external electromagnetic (EM) field. The incompressible MHD model is used in many engineering and scientific disciplines from the development of nuclear fusion as a sustainable energy source to the study of space weather and solar physics. Many interesting MHD systems exhibit the phenomenon of turbulence which remains an elusive problem from all scientific perspectives. This work focuses on the computational perspective and proposes techniques that enable the study of systems involving MHD turbulence.

Direct numerical simulation (DNS) is not a feasible approach for studying MHD turbulence. In this work, turbulence models for incompressible MHD were developed from the variational multiscale (VMS) formulation wherein the solution fields were decomposed into resolved and unresolved components. The unresolved components were modeled with a term that is proportional to the residual of the resolved scales. Two additional MHD models were developed based off of the VMS formulation: a residual-based eddy viscosity (RBEV) model and a mixed model that partners the VMS formulation with the RBEV model. These models are endowed with several special numerical and physics features. An important aspect of the new VMS models is the formal consistency of the method. That is, since the model is residual based, the continuous exact solution satisfies the weak form operators that implement the VMS cross-couping fluxes and the dissipative terms. Physically, the new models are able to capture desirable MHD physics such as the inverse cascade of magnetic energy and the subgrid dynamo effect. Moreover, we show that the models are self-adapting isofar as they are able to capture nonuniversal energy spectra in MHD turbulence without parameter tuning.

The models were tested with a Fourier-spectral numerical method and the finite element method (FEM). The primary test problem was the Taylor-Green vortex. Results comparing the performance of the new models to DNS were obtained. The performance of the new models was compared to classic and cutting-edge dynamic Smagorinsky eddy viscosity (DSEV) models. The new models typically outperform the classical models.