Subcell artificial viscosity shock capturing versus slope limiting for shallow water equations using a positivity preserving high order well-balanced Runge-Kutta discontinuous Galerkin scheme

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ABSTRACT

The shallow water equations have been widely used to simulate different hydrological models such as water channels, rivers, coastal plains and flooding. An important outcome arising in these simulations is the appearance of dry areas which cannot be described by standard numerical methods. Another complex feature that commonly appears in this kind of simulation is the existence of discontinuous solutions as observed in dam-break problems and breaking wave formation. In this work, we present a novel approach to treating shock discontinuities and drying patches simultaneously in a time explicit high order discontinuous Galerkin finite element (DG-FEM) discretization of the one-dimensional shallow water equations. The central idea conveyed by the numerical algorithm is the combination of subcell shock capturing strategy with positivity preserving of water height. Numerical stability for discontinuous solutions is achieved by adding small amounts of artificial viscosity only where spurious oscillations are detected [3]. Non-negativity of water height is ensured by a simple positivity preserving limiter that reconstructs linear or constant elementwise solutions with minimum height tolerance over dry bed regions [4, 2]. The suitability of the new algorithm with respect to theoretical results has previously been proved in another publication [1]. For the present paper, we compared the subcell shock capturing strategy with two kinds of limiters, namely, generalized and hierarchical slope limiters [5, 6] while maintaining positive the water height. Numerical results demonstrated that subcell approach with very high order approximation (polynomial degree up to 10) is more accurate than limiters for a fixed number of degrees of freedom.

REFERENCES

[1] Almeida, J.L.S., Silva, C.A.C. and Nogueira Jr., A.C. Positivity preserving high order well-balanced Runge-Kutta discontinuous Galerkin scheme with artificial viscosity based shock capturing for shallow water equations: the one dimensional case *Proceedings of the HONOM 2017 – ECCOMAS Thematic Conference on High Order Nonlinear Numerical Methods for Evolutionary PDEs*, Stuttgart, Germany, March 27-31, 2017 (to appear).

[2] Gandham, R., Medina, D. and Warburton, T. GPU Accelerated Discontinuous Galerkin Methods for Shallow Water Equations *Communications in Computational Physics*, Vol. **18(1)**, pp. 37–64, (2015).

[3] Klöckner, A. and Warburton, T. Viscous Shock Capturing in a Time-Explicit Discontinuous Galerkin Method. *Mathematical Modelling of Natural Phenomena*, Vol. **6(3)**, pp. 57–83, (2011).

[4] Xing, Y., Zhang, X. and Shu, C.W. Positivity preserving high order well balanced discontinuous Galerkin methods for the shallow water equations *Advances in Water Resources*, Vol. **33(131)**, pp. 1476–1493, (2010).

[5] J.S. Hesthaven, T. Warburton, *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications*. Springer, 2008.

[6] Krivodonova, L. Limiters for high-order discontinuous Galerkin methods. *Journal of Computational Physics*, **226**, pp. 879–896, (2007).