Eliminating oscillations near discontinuities using a non-linear Petrov-Galerkin method in Banach spaces

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ABSTRACT

If sharp features such as interior or boundary layers are present in the solution of a PDE, the standard Galerkin approach based on a weak formulation in the Hilbert space H^1 frequently leads to highly oscillatory numerical approximations and thus has to be supplemented by suitable stabilisation techniques.

The idea we are pursuing is to consider the approximation problem as a residual minimisation problem in L^p-type Sobolev spaces, cf., [1], with $1 < p < \infty$. We then apply a non-standard, non-linear Petrov-Galerkin discretisation, proposed in [2], that is applicable to reflexive Banach spaces such that the space itself and its dual are strictly convex. Similar to discontinuous Petrov-Galerkin methods [3], this method is based on employing optimal test functions. Replacing the intractable optimal test space by a tractable approximation gives rise to a non-linear inexact mixed method for which optimal a priori estimates hold.

A key advantage of considering a more general Banach space setting is that the oscillations in the numerical approximation vanish as p tends to 1, as we will demonstrate using a few simple numerical examples.

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