Convergence analysis of the Dirichlet-Neumann iteration for unsteady thermal fluid structure interaction

Azahar Monge*, **Philipp Birken**

Centre for Mathematical Sciences
Lund University
Box 118, 22100 Lund, Sweden
e-mail: azahar.monge@na.lu.se, web page: http://www.maths.lu.se/staff/azahar-monge/

ABSTRACT

The efficient simulation of thermal interaction between fluids and structures is crucial in the design of many industrial products, e.g. thermal anti-icing systems of airplanes or the cooling of rocket thrust chambers. The latter consists of a system with two related thermal interactions between fluids and structures. On one hand, the thermal interaction between the hot gas coming out from the combustion chamber and the structure recovering the nozzle. On the other hand, the thermal interaction between the cooling fluid and the structure.

Unsteady thermal fluid structure interaction is modelled using two partial differential equations describing a fluid and a structure on different domains. The equations are coupled at an interface to model the heat transfer between fluid and structure where we have jumps in the material coefficients across the interface. The standard algorithm to find solutions of the coupled problem is the Dirichlet-Neumann iteration, where the PDEs are solved separately using Dirichlet-, respectively Neumann boundary conditions with data given from the solution of the other problem. Previous numerical experiments [1] show that this iteration is fast, and although the iteration has been analyzed and a convergence condition is given in [4], the convergence rates have not been computed.

We consider the transmission problem as a basic building block in thermal fluid structure interaction. A convergence analysis for the semi-discretized equations of the thermal transmission problem was provided in [2]. However, our numerical results for the fully-discretized case are not completely covered by this analysis, and therefore, we propose a complementary analysis for this case.

Our analysis predicts very accurately the behaviour of the Dirichlet-Neumann iteration. Moreover, we can also estimate the asymptotic behaviour of the convergence rates when both the spatial mesh size and the stepsize tend to 0. Numerical results are presented to illustrate the analysis.

REFERENCES

[1] Birken, P. and Gleim, T. and Meister, A. and Kuhl, D. Fast solvers for unsteady thermal fluid structure interaction. *Int. J. Numer. Meth. Fluids.* (2015) **79**(1):16–29.

[2] Henshaw, W.D. and Chand, K.K. A composite grid solver for conjugate heat transfer in fluid-structure systems. *J. Comp. Phys.* (2009) **228**:3708–3741.

[3] Quarteroni, A. and Valli, A. *Domain decomposition methods for partial differential equations.* Oxford Science Publications (1999)