# Variational multi-scale finite element approximation of the compressible Navier-Stokes equations

**Camilo Bayona*, Joan Baiges and Ramon Codina**

International Center for Numerical Methods in Engineering (CIMNE)
Universidad Politécnica de Cataluña
Campus Norte UPC, 08034 Barcelona, Spain
*e-mail: cbayona@cimne.upc.edu

## ABSTRACT

In this work we apply the Variational Multi-Scale (VMS) framework to the finite element approximation of the compressible Navier-Stokes equations written in conservation form. Even though this formulation is relatively well known, we incorporate some particular features of the VMS framework that we have applied in other flow problems. Since there are different ways to define the subscales, three different attributes are studied in this work. The first attribute is the definition of the space of the subscales, which can be either the space of finite element residuals, or the space orthogonal to the finite element space [1]. The second attribute is the inclusion of the subscales in all the non-linear terms of the problem [2]. The third attribute is the inclusion of the transient term of the subscales equation [3]. In addition to these definitions, in our stabilized formulation we aim to give a systematic way to design the matrix of algorithmic parameters from the perspective of a Fourier analysis.

Because the subgrid stabilization method works in the streamline direction, we implement an anisotropic shock capturing method that keeps the diffusion unaltered in the direction of the streamlines, but modifies the crosswind diffusion. We also propose to calculate the artificial shock capturing diffusivity by using the orthogonal projection onto the finite element space of the gradient of the solution, instead of the common residual definition.

Subsonic and supersonic numerical examples are discussed. It has been found that including the orthogonal, dynamic, and the non-linear subscales improves the accuracy of the compressible formulation. The results indicate a less dissipative behaviour of the orthogonal subscales compared to the finite element residual definition for the space of subscales. Because the anisotropic method adds the artificial diffusivity only in the crosswind direction, the non-linearity introduced by the shock capturing method has less effect in the convergence behavior to the steady state.

## REFERENCES

[1] R. Codina. Stabilization of incompressibility and convection through orthogonal sub-scales in finite element methods. *Comput. Methods. Appl. Mech. & Eng.*, 190(13):1579–1599, 2000.

[2] R. Codina. A stabilized finite element method for generalized stationary incompressible flows. *Comput. Methods. Appl. Mech. & Eng.*, 190(20):2681–2706, 2001.

[3] R. Codina, J. Principe, O. Guasch, and S. Badia. Time dependent subscales in the stabilized finite element approximation of incompressible flow problems. *Comput. Methods. Appl. Mech. & Eng.*, 196(21):2413–2430, 2007.