FEM-type Approach to Boundary Condition Satisfaction in Vortex Element Method for 3D Flow Simulation

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ABSTRACT

Vortex methods are well-known alternative to traditional mesh methods in CFD for incompressible flow simulation. They are especially efficient in engineering practice for numerical simulation in unsteady coupled FSI-problems. One of the most important problems in vortex methods is body conditions satisfaction on streamlined body surface.

Traditionally body surface is simulated by double layer of unknown intensity and the so-called ‘panel’ method [1, 2] is used to solve the corresponding integral equation. The surface is discretized into polygonal panels, and double layer intensity in vortex methods normally is assumed to be constant on every panel. Such panels influence the flow in the same way as closed vortex framework placed on the boundary of the panel. Such class of vortex methods is sometimes called ‘vortex lattice’ methods, because all the frameworks have common legs and in aggregate they cover the surface. The disadvantage of this approach is that 1) the integral equation is hypersingular and existing numerical schemes for it has low accuracy; 2) the velocity as well as pressure field is not smooth near the surface, especially in boundary layer.

In the present paper the alternative approach is developed, it is based on the idea of streamlined surface simulation by vortex layer instead of double layer and another way to boundary condition satisfaction [3]. In such case instead of vortex lattice we deal with ‘vortex mesh’, i.e. vortex sheet distributed over the surface. In order to solve the corresponding integral equation the basic ideas of FEM method are suitable: the solution is assumed to be combination of the shape functions. The most suitable approach is discontinuous Galerkin method, because the solution is not required to be continuous and smooth.

In the research original numerical DG-type scheme is developed for vortex sheet intensity computation. Semi-analytical expressions are obtained for influence matrix coefficients, which are the improper double integrals of unbounded functions over the panels. Analytic singularity exclusion makes it possible to compute the mentioned integrals with high accuracy and raise significantly the accuracy of vortex methods in the whole.

The results of model problems solution show that the developed approach and numerical algorithm allow to simulate the flow around bodies with complicated geometry, use triangular unstructured surface meshes and arbitrary refine the mesh.

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