Characteristics-mixed FEMs for miscible displacement in porous media

Weiwei Sun
City University of Hong Kong

Abstract

We study the following miscible displacement system modeling an incompressible flow in porous media

$$\Phi\frac{\partial c}{\partial t} - \nabla\cdot(D(\mathbf{u})\nabla\mathbf{c}) + \mathbf{u}\cdot\nabla\mathbf{c} = \mathbf{c_1 q^I} - \mathbf{c q^I}, \tag{1}$$

$$\nabla\cdot\mathbf{u} = \mathbf{q^I} - \mathbf{q^P}, \tag{2}$$

$$\mathbf{u} = -\frac{\mathbf{k(x)}}{\mu(\mathbf{c})}\nabla\mathbf{p}, \tag{3}$$

for $t \in [0, T]$, with the initial condition $c(x, 0) = c_0(x)$, by a class of characteristics FEMs. Here, c represents the concentration of one of the fluids. Φ denotes the porosity of the medium, q^I and q^P are given injection and production sources, c_1 is the concentration of the injection source, $D(\mathbf{u}) = [\mathbf{D_{ij}}(\mathbf{u})]_{\mathbf{d\times d}}$ is the diffusion-dispersion tensor (see [1] for details), $k(x)$ is the permeability of the medium and $\mu(c)$ is the concentration-dependent viscosity of the fluid mixture.

The method of characteristics type is especially effective for convection-dominated diffusion problems. A modified method of characteristics (MMOC) with both finite difference and finite element approximations was proposed by Douglas and Russell [2] for linear convection-dominated diffusion problems and by Russell for the nonlinear miscible displacement equations in both two and three dimensional spaces. Later, further developments on characteristics methods with many other spatial approximations have been studied [3, 4] and methods have been applied and analyzed for many linear and nonlinear parabolic PDEs from various engineering applications.

Due to the nature of characteristic temporal discretization, the method often allows one to use a large time step in many practical computations, while all previous theoretical analyses always required certain restrictions on the time stepsize. In this talk, we present our recent work on establishing unconditionally optimal error estimates for a modified method of characteristics with a mixed finite element approximation to the miscible displacement problem in $\mathbf{R}^d$ ($d = 2, 3$). For this purpose, we introduce a characteristic time-discrete system. We prove that the L^2 error bound of the fully discrete method of characteristics to the time-discrete system is τ-independent and the numerical solution is bounded in $W^{1,\infty}$-norm unconditionally. With the boundedness, optimal error estimates are established in a traditional manner. Numerical results confirm our theoretical analysis and clearly show the unconditional stability.

References

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