A Mixed DG Framework for the Second-Order Boltzmann-Based Hypersonic Rarefied and Low-Speed Microscale Gas Dynamics

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ABSTRACT

A mixed-type discontinuous Galerkin (DG) framework based on the second-order Boltzmann kinetic equation [1] is presented for hypersonic rarefied and low Mach number microscale gas flows. An explicit modal DG scheme on unstructured triangular/tetrahedral meshes is considered for solving the multi-dimensional conservation laws in conjunction with non-Newtonian type nonlinear coupled constitutive relations (NCCR) arising from the high degree of thermal non-equilibrium [2-5].

The verification of the DG scheme was achieved by comparing numerical solutions of a stiff problem of the shock structure with corresponding analytical solutions for all Mach numbers; not only the inner shock structure profiles but also the differential and integral quantities of the shock structure like the thickness, the asymmetry, and the density-temperature separation distance. In addition, the DG scheme was validated by investigating the hypersonic rarefied and low-speed microscale gas flows past cylinder, sphere, and blunt body. Further, a supersonic rarefied gas flow over airfoil and associated drag polar were studied by the DG scheme of the second-order macroscopic model of the Boltzmann equation combined with Langmuir velocity slip and temperature jump models.

Moreover, a single program multiple data (SPMD) parallel model using a message-passing-interface (MPI) library was employed to parallelize the mixed modal DG method. The parallelization of the NCCR solver in the DG framework was shown to benefit from a drastic cost reduction in comparison with the Navier-Stoke-Fourier solver, due to its super-parallel performance whose origin can be traced to the nonlinear dependence of the number of iterations of the NCCR solver on the flow structure.

Finally, the issues of developing proper limiters and viscous treatment methods for the DG scheme in wide range of flow regimes defined by various Mach and Knudsen numbers are discussed.

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