High order $H(\mathrm{div})$-non conforming Hybrid Discontinuous Galerkin Methods for steady and unsteady incompressible flows

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ABSTRACT

In this work we consider different high order discretization methods for steady and unsteady incompressible flows. Starting with the stiff linear part of the steady Stokes problem we first use $H(\mathrm{div})$-conforming finite elements to garantuee point wise exactly divergence-free velocity solutions leading to local conservation and pressure robust error estimates, furthermore we use a hybridized version for an efficient implementation. In a second step we use a projection operator for the (hybridized) tangential component which does not interfere the normal continuity, thus still leads to an $H(\mathrm{div})$-conforming method but reduces the number of globally coupled unknowns. In a final step we also break the normal continuity hence we use $H(\mathrm{div})$-non conforming Finite Elements. This leads to weakly divergence-free velocity solutions. We show that the method still satisfy an optimal error estimation, and providing a simple reconstruction operator which maps $H(\mathrm{div})$-non-conforming velocity fields to $H(\mathrm{div})$-conforming ones, we also show that this leads to an optimal *pressure robust* error estimation. Furthermore the reconstructed velocity field is used for a discretization of the Navier-Stokes equations based on an operator-splitting time integration scheme. We present the methods, discuss the analysis and present numerical benchmarks to demonstrate the efficiency and accuracy of the discretizations.

REFERENCES

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