Two-phase compressible-incompressible problems: numerical methods and optimal control

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ABSTRACT

This talk deals with two-phase compressible-incompressible flows. Springing from the industrial interest in effectively controlling the production process of a metal foam (that is, a structure consisting of solid metal with gas-filled pores), we formulated an optimal control problem on the evolution of the gas bubbles within the liquid metal during such process. The final position of the gaseous inclusions, as a matter of fact, is of great importance in determining the physical and mechanical properties of the resulting foam.

The two-phase model used is a Finite Element recast of the temperature-dependent Navier-Stokes formulation proposed in [2] and it is based on the Low-Mach approximation for the flow model in the gaseous phase. The range of Reynolds numbers in our applications justify this assumption. The same set of equations is therefore valid for both phases (albeit with different coefficients) so that each flow quantity is defined as a single function on the whole domain.

Due to the highly-coupled nature of the model and to the (possibly) large difference in physical properties among the two phase, a proper solution scheme had to be derived. A highly-general C++ library based on FEniCS was developed to implement and solve the control problem. Some numerical results (on both direct and control problems) are shown and the controllability of the system through the imposition of a optimal Dirichlet boundary condition on the temperature is discussed.

REFERENCES

[1] Alazard, T. Low Mach Number Limit of the Full Navier-Stokes Equations, *Archive for Rational Mechanics and Analysis*, n. 1, pp. 1–73, Vol. 180, (2005).

[2] Daru, V., Le Quéré, P., Duluc, M.-C. and Le Maître, O. A numerical method for the simulation of low Mach number liquidgas flows. *Journal of Computational Physics*, n. 23, pp. 8844–8867, (2010).

[3] Gunzburger, Max D. Perspectives in Flow Control and Optimization. *Siam*, Vol. 5 (2003)

[4] Logg, A., Mardal, K.-A. and Wells, G. Automated solution of differential equations by the finite element method: The FEniCS book, *Springer Science & Business Media*, Vol. 84 (2012).

[5] Tröltzsch, F. Optimal Control Of Partial Differential Equations, *Graduate studies in mathematics*, Vol. 112 (2010).