Mesh Refinement for Quasilinear Two-Grid Discontinuous Galerkin Finite Element Methods with Polygonal Meshes

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ABSTRACT

In this talk, we study mesh refinement for the two-grid discontinous Galerkin finite element method for strongly monotone quasilinear PDEs using general polygonal elements. Two-grid methods, cf. [1, 5, 6], first approximate an underlying nonlinear problem on a coarse finite element partition of the computational domain and then on the basis of this coarse grid approximation compute a linearized variant of the discrete problem on a finer mesh. It is necessary to design the two meshes such that the linear solve is performed on a finer mesh. In previous articles [3, 4] we have studied the *hp*-version adaptive mesh refinement of both coarse and fine meshes for discontinuous Galerkin two-grid methods for strongly monotone quasilinear PDEs based on the *a posteriori* error bound.

Recent developments [2] have studied the use of general polygonal and polyhedral elements for the standard discontinuous Galerkin finite element method. One method of generating meshes of general polygonal elements is to agglomerate a set of standard elements into a polygonal element. Using this agglomeration technique it is possible to create a fine mesh of general simplex elements and a coarse mesh generated by agglomerating the fine mesh elements. Using these meshes it is possible to run a two-grid method with a given mesh of simplex elements and an automatically generated mesh of general agglomerated polygonal elements. In this talk we extend the work in our existing two-grid mesh refinement articles, which uses only simplex, quadrilateral or hexahedral elements, to a two-grid method with an agglomerated coarse mesh and discuss an automatic mesh refinement algorithm for the two meshes.

REFERENCES

[1] C. Bi and V. Ginting. Two-grid discontinuous Galerkin method for quasi-linear elliptic problems. *J. Sci. Comput.*, 49(3):311–331, 2011.

[2] A. Cangiani, E. H. Georgoulis, and P. Houston. *hp*-version discontinuous Galerkin methods on polygonal and polyhedral meshes. *Math. Models Methods Appl. Sci.*, 24(10):2009–2041, 2014.

[3] S. Congreve, P. Houston, and T. P. Wihler. Two-grid *hp*-version discontinuous Galerkin finite element methods for second-order quasilinear elliptic PDEs. *J. Sci. Comput.*, 55(2):471–497, 2013.

[4] S. Congreve and P. Houston. Two-grid *hp*-version discontinuous Galerkin finite element methods for quasi-Newtonian flows. *Int. J. Numer. Anal. Model.*, 11(3):496–524, 2014.

[5] J. Xu. A new class of iterative methods for nonselfadjoint or indefinite problems. *SIAM J. Numer. Anal.*, 29:303–319, 1992.

[6] J. Xu. A novel two-grid method for semilinear elliptic equations. *SIAM J. Sci. Comput.*, 15 (1):231–237, 1994.