On the Reflection, Transmission, Coupling and Damping of Non-Plane Acoustic Modes by Resonator Rings

A. Cárdenas Miranda and W. Polifke
Lehrstuhl für Thermodynamik, TU München
cardenas@td.mw.tum.de

Abstract

Due to a possible feedback between heat release and acoustics, rocket engines are prone to thermo-acoustic instabilities. Because energy density and efficiency of modern engines are extremely high, the resulting high frequency oscillations can lead to severe anomalies of motor performance and even to the destruction of the thrust chamber in a very short time. Acoustic cavities, so called resonators, are commonly attached to the combustion chamber in order to increase the acoustic losses and thus the stability of the engines. However, their stabilizing influence is not totally clarified yet, and thus, their effective design under operation conditions is not a simple task.

As a first approximation during the design process, the stabilizing influence of resonator rings can be studied by means of linear acoustics. The resonator ring is here modeled as a cylindrical duct with characteristic reactive and dissipative boundary condition at the cylinder shell, commonly referenced as "soft-wall" duct. The characteristic impedance is build as a parallel arrangement of simple cavity mouth impedances. While the real valued or dissipative part of the impedance can directly promote stability, the influence in combination with the complex valued or reactive part is not nescessarily trivial.

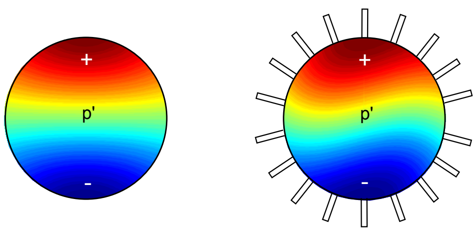

Figure 1: Mode shape of first tangential mode with and without resonator cavities.

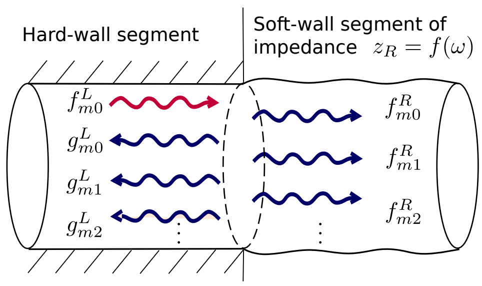

Figure 2: Mode scattering and coupling at a discontinuity of shell impedance in a cylindrical duct.

In order to describe the propagation of non-plane transversal forward f_{mn} and backward g_{mn} traveling waves through resonator rings, the three dimensional acoustic field in such a "soft-wall" duct segment is calculated by solving the convective wave equation in the frequency domain. Due to the dissipation represented by the real valued part of the impedance boundary condition, the characteristic amplitudes of the waves decrease while traveling through the ring. However, the reactive or complex valued part of the impedance changes the transversal mode shape of the waves, see for example Fig. 1. At connecting planes of the resonator ring, a forward traveling wave f^L_{m0} of tamgential order m and radial order 0 get thus partially transmitted f^R_{m0}, f^L_{m1}, f^L_{m2}, ... and partially reflected g^R_{m0}, g^R_{m1}, g^R_{m2}, ... Furthermore, so called mode coupling occurs and the waves scatter into different radial mode orders 1,2,3,... as sketched in Fig. 2.

In this paper, a semi-analytical model describing the propagation of three dimensional non-plane waves through resonator rings used in rocket thrust chambers is presented and extended to account for scattering and mode coupling at connecting planes and discontinuities. The method for this so called "mode-matching" technique is based on the conservation of mass and momentum in an integral manner over the connecting plane using a weak form of the linearized conservation equations following the Galerkin approach. The derivation of the method is given and subsequently validated in a generic test case of a cylindrical duct with a sudden jump in shell impedance. The accuracy of the mode matching method is proved by comparison of acoustic pressure and velocity profiles at both sides of the discontinuity. Depending on the impedance boundary condition, a different number of radial mode orders is needed for the matching.

The method is then applied to characterize the performance of generic resonator rings for the stabilization of rocket thrust chambers. The reduction of an externally imposed sound pressure level through such a ring can be confirmed with the here presented method. Furthermore, when taking the mode scattering and coupling at discontinuities into account, the reduction of sound pressure level can be noticeable stronger. The reasons for this behavior are explained in the paper. When scattered into higher order modes, the traveling waves decay exponentially along the hard wall segments at frequencies below cut-on. Thus, the reactive part of the impedance boundary condition can contribute in the dissipation of acoustic energy, by shifting acoustic energy into evanescent higher order modes. This behavior can be exploited to increase the performance of resonator rings.

The here presented research has been founded by the German Research Foundation (DFG) in the framework of the Sonderforschungsbereich Transregio 40 that deals with the technological foundations for the design of thermally and mechanically highly loaded components of future space transportation systems.