Computing the vibrations of polygonal panels under distributed random excitation using the image source method

Jacques Cuenca

LMS International, Interleuvenlaan 68, 3001 Leuven, Belgium

Flexural vibrations are responsible for noise radiation and damage in the airframe of aircraft. The most commonly used methods in academia and industry for predicting such vibrations are finite element analysis (FEA) and statistical energy analysis (SEA). Owing to their underlying hypotheses and numerical constraints, these methods are respectively limited to low and high frequencies. However, the so-called mid-frequency range is characterised by interactions between short- and long-wavelength phenomena and thus requires the development of approaches that do not rely on purely deterministic or statistical hypotheses [1].

This paper presents a method for computing the vibrations of polygonal plates subjected to a spatially-distributed random excitation. The method is developed within the scope of assessing the vibratory impact of turbulent flow on the airframe of aircraft in the mid-frequency range. The image source method consists in representing wave reflection on a given boundary of a structure by means of the mirror image of the source and the reflection properties of the boundary. Thus, the position of the image source is obtained by mirror symmetry of the original source with respect to the boundary. In the case of a finite structure such as a polygonal plate, an infinite number of reflections successively occur at the different boundaries [2, 3]. The response of the polygonal plate to a given source of vibrations can thus be represented as a superposition of the contributions from an infinite number of image sources. The latter are obtained by performing successive symmetries of the original source and consecutively applying the reflection properties of the boundaries at each reflection. Figure 1 shows a point-driven polygonal plate and the first image sources.

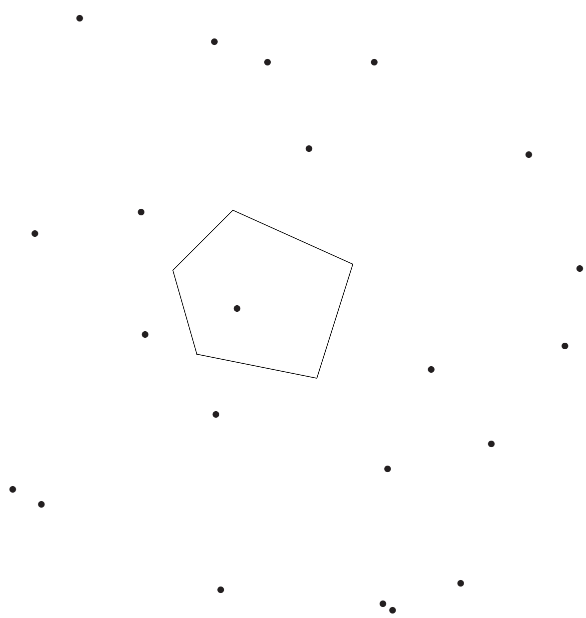

Figure 1: First image sources for a polygonal plate excited by a point source.

The Green's function of the plate for a source at $\mathbf{r}_0$ and an observation point at $\mathbf{r}$ can be written as

$$G_\Omega(\mathbf{r}, \mathbf{r}_0; k_f) = G_\infty(\mathbf{r}, \mathbf{r}_0; k_f) + \sum_{s=1}^{\infty} G_s(\mathbf{r}, \mathbf{r}_s; k_f), \tag{1}$$

where G_∞ is the Green's function of an infinite plate with the same material properties, G_s are the image source contributions with $\mathbf{r}_s$ the image source positions and k_f is the flexural wavenumber. The out-of-plane displacement of a polygonal plate subjected to an arbitrary source can be written as

$$w(\mathbf{r}, \mathbf{r}_0; k_f) = w_0(\mathbf{r_0}; k_f) * G_\Omega(\mathbf{r}, \mathbf{r}_0; k_f), \tag{2}$$

where w_0 is the displacement of the source and $*$ denotes convolution product on variable $\mathbf{r}_0$. The source term w_0 contains all the information on the excitation, while the Green's function G_Ω entirely characterises the structure. Using this formulation, the response of the plate can be simulated for an arbitrary spatially-distributed source. In the present study, the excitation is considered over the entire plate or over a finite region, as can be the case for a turbulent boundary layer [4]. Furthermore, the randomness in the excitation can be modelled by means of a probability density function.

Outline

First, the theory of the image source method is developed for a point source and then a spatially-distributed source. Then, the reciprocity of the Green's function is employed in order to obtain the response to a spatially-distributed random excitation. The domain of applicability of the method is determined and it is then applied to practical cases. Several spatial distributions of the excitation are examined and the effect of the probability density function of the excitation is studied.

References

[1] B.R. Mace, K. Worden, and G. Manson. Uncertainty in structural dynamics. *Journal of Sound and Vibration*, 288(3):423–429, 2005.

[2] J. Cuenca, F. Gautier, and L. Simon. The image source method for calculating the vibrations of simply supported convex polygonal plates. *Journal of Sound and Vibration*, 322(4-5):1048–1069, 2009.

[3] J. Cuenca, F. Gautier, and L. Simon. Harmonic Green's functions for flexural waves in semi-infinite plates with arbitrary boundary conditions and high-frequency approximation for convex polygonal plates. *Journal of Sound and Vibration*, 331(6):1426–1440, 2012.

[4] H. Schlichting and K. Gersten. *Boundary Layer theory.* Springer, 2004.