Optimality of Standard Flight Procedures of Commercial Aircraft

Antonio Franco*, Alfonso Valenzuela†, and Damián Rivas‡

Escuela Técnica Superior de Ingeniería, Universidad de Sevilla, Seville, 41092, Spain

Abstract

Trajectory optimization is, from the operational point of view, a subject of great importance in air traffic management (ATM), that aims at defining optimal flight procedures that lead to energy-efficient flights. Despite their theoretical interest, optimal solutions may not be flyable according to present-day air traffic procedures and regulations. However, they represent best performance and can be used to assess the optimality of flight procedures commonly used in practice. In this work, the following standard procedures are analyzed (corresponding to different flight phases, with different performance indices):

1) CAS-Mach climb with fixed engine rating, with the objective of minimizing fuel consumption;

2) constant-Mach cruise at constant altitude, with the objective of minimizing fuel consumption in the case of fixed arrival time; and

3) constant-CAS unpowered descent, with the objective of maximizing range.

The optimal solutions are obtained using the theory of singular optimal control. In all those problems to be solved, there is one control variable which appears linearly in the equations of motion, as well as on the performance indices to be optimized. As a consequence, the Hamiltonian of the problem is also linear on the control variable, which leads to a *singular optimal control problem* [1].

Let H be the Hamiltonian of the problem and u the control variable. The derivative H_u is called the switching function. The optimal control problem reduces to finding the optimal control u^* that minimizes H, satisfying the equations of motion. In general, u^* is determined by the necessary condition for optimality $H_u = 0$, but in the problems to be solved the function H_u does not depend on u (hence, $H_{uu} = 0$, reason why the problem is called singular). The singular control now follows from the condition $\ddot{H}_u = 0$ (the function $\ddot{H}_u$ happens to be linear in u). The corresponding optimal path lies on a singular manifold in the state space called *singular arc* (the singular arc is in

*Assistant Professor, Department of Aerospace Engineering; antfranco@us.es.

†Assistant Professor, Department of Aerospace Engineering; avalenzuela@us.es.

‡Professor, Department of Aerospace Engineering; drivas@us.es.

fact the locus of possible points in the state space on which optimal paths can lie). Integration along the optimal path leads to optimum performance index.

In this work it is assumed that the initial and final points of the path are given. In that case, the optimal path is formed, in general, by three arcs: one to go from the initial point to the singular arc, the singular arc, and a final arc to go from the singular arc to the final point. The initial and final arcs are defined by the control being at its maximum or minimum value; this type of optimal control is called bang-singular-bang [1].

The standard flight procedures are defined in terms of flight segments commonly flown by airlines (constant CAS, constant Mach, constant altitude, etc.), and are optimized using parametric optimization theory [2]. Once the optimum values of the parameters (CAS, Mach, altitude, etc.) are obtained, the optimized standard procedures are compared with the optimal trajectories, and their optimality assessed.

In the case of minimum-fuel climb with fixed engine rating, the CAS-Mach procedure is formed by four segments, all of them with given engine rating: horizontal acceleration at the initial altitude h_i, from the initial speed V_i to the climb CAS (CAS_c); climb with constant CAS (CAS_c), from h_i to the transition altitude h_t (at which the climb Mach M_c is reached); climb with constant Mach (M_c), from h_t to the final altitude h_f; and horizontal acceleration at altitude h_f, from M_c to the final speed V_f. The results show that the CAS-Mach procedure is not close to optimal, in the sense that the optimal speed law during the first part of the climb is not at contant CAS. However, the integral performance of the optimized CAS-Mach procedure is very close to optimal, that is, the fuel consumption, the flight time and the horizontal distance travelled are very close to the optimum values.

In the case of minimum-fuel cruise at constant altitude with fixed arrival time, the constant-Mach procedure is formed by three segments: horizontal acceleration/deceleration from the initial speed V_i to the cruise speed V_c, with maximum cruise/idle engine rating; horizontal cruise with constant speed V_c; and horizontal acceleration/deceleration from V_c to the final speed V_f, with maximum cruise/idle engine rating. The results show that the optimized constant-Mach cruise is very close to optimal, in the sense that the fuel consumption is very close to the optimum value.

In the case of maximum-range unpowered descent, the constant-CAS procedure is formed by 3 segments, all of them with zero thrust: horizontal deceleration at the initial altitude h_i, from the initial speed V_i to the descent CAS (CAS_d); descent with constant CAS (CAS_d), from h_i to the final altitude h_f; and horizontal deceleration at altitude h_f, from CAS_d to the final speed V_f. It is shown that the optimized constant-CAS procedure is very close to optimal, and also, as a consequence, that the range and flight time are very close to the optimum values.

Results are presented for a model of a Boeing 767-300ER (a typical twin-engine, wide-body, transport aircraft), with compressible aerodynamics and engine model dependent on speed and altitude.

References

[1] Bryson, A.E., and Ho, Yu-Chi, *Applied Optimal Control*, Hemisphere Publishing Corporation, 1975.

[2] Fletcher, R., *Practical Methods of Optimization*, John Wiley & Sons, 1987.