Gradient Calculation for Arbitrary Parameterizations via Volumetric NURBS: The Control Box Approach.

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Keywords— CAD-based optimization, aerodynamic shape optimization, NURBS, design variables, control box.

I. Introduction

THE gradient-based optimization strategy, combined with accurate flow simulations and the use of the adjoint formulation to efficiently calculate the gradients, has been proved a promising approach for the improvement of the aerodynamic performance or aircraft designs [1]. However, the deployment of this technology in an industrial environment still faces practical limitations.

One of the main practical issues faced by the optimization process is to recover the geometric description after the optimization, in order to easily couple with additional tools and for post-processing. This geometric description usually comes from Computer Aided Design (CAD) software, such as IGES file format, which usually employs Non-Uniform Rational B-Splines (NURBS) to represent the surface skin of the geometry. In this context, there are two approaches: the so called CAD optimization techniques that involve the CAD geometric description throughout the whole optimization loop, and CAD-free techniques that recover the CAD format from the optimized grid as a post-processing. The main shortcoming for this second approach are dealing with intersections between different components of the geometry [2] and numerical errors induced during the CAD reconstruction.

Another main practical issue is to link the design variables to the sensitivities provided by the adjoint solution. Some approaches are specific for one particular aircraft component that may implicitly contains restrictions of the design or represent physical properties. For example, in the PARSEC parameterization introduced by Sobieczky [3], the shape of an airfoil is described by geometric parameters related to physical properties, such as leading edge radius, thickness ratio or trailing edge angle, among others. Other design variables employs generic geometric descriptions, such as b-splines [4], which allow off-the-book designs, but ignore any engineering information and therefore are not intuitive for the designer. Kulfan [5] suggested a technique that combines a class function and a shape function, defined by Bernstein polynomials; however, while it fulfils the four Samareh conditions [6], it is not able to efficiently cope with local changes. In addition, the selection of appropriated design variables for optimization is still an open issue. However, a link between the computational grid and the CAD geometric description with the chosen design variables is required, which may involve complex mathematical equations or are simply inaccessible data buried in closed source binaries.

This paper proposes a generic methodology to link any design variable with both the CAD geometric description and the computational grid through an innovative concept: the control box.

II. Calculation of the Gradients

The use of the adjoint methodology has been introduced during the last decade as an efficient method for calculating the gradients involving a large number of design variables; as it requires a single adjoint solution for each cost function (e.g. drag, lift, and momentum).

For aerodynamic drag and lift optimization problems, the functional is defined as [7]:

$$J = \int_S C_p (\vec{n} \cdot \vec{d}) ds \tag{1}$$

M. J. Martin is a PhD student at UPM, who also works (subcontracted) in the Fluid Dynamics Branch of INTA where C_P is the pressure coefficient, n is the normal to the boundary surface S, and d is the force direction vector defined by the angle of attack α and the sideslip angle β. By considering the local surface sensitivity δj in the local normal direction, obtained from the adjoint solution and the geometric sensitivities δx, the gradients of the functional with respect to design variables are calculated as [7]:

$$\delta J = \int_S \delta j \; (\delta x \cdot \vec{n}) ds \tag{2}$$

The above formulation considers tangential deformations of the surface negligible. However, edges and strong changes of the curvature in the geometry could require a special treatment that includes the tangential derivatives [8].

In this paper, the key terms are the geometric sensitivities δx. From a mathematical perspective, if D is a function that represents a design space, the sensitivities are calculated as the derivative of the vertex coordinates upon variations of the design parametric coordinates λ_i.

$$x = D(\lambda_i); \delta x = \frac{\partial x}{\partial D} = \frac{\partial D(\lambda_i)}{\partial \lambda_i} \tag{3}$$

Therefore, in order to obtain the geometric sensitivities, the parametric coordinates λ_i are required for all surface vertices of the computational grid: $\lambda_i = D^{-1}(x)$, which are not always easy to obtain. Some design variables are specifically chosen to deal with this problem; by employing Free Form Deformation [9], the parametric coordinates of the Bernstein polynomials can be directly obtained from the Cartesian coordinates by a linear transformation.

III. Control Box

In this approach, if the CAD is also involved, the chain rule is applied to the geometric sensitivities. The geometry is represented by a NURBS surface, defined by a grid of control points S.

$$\delta x = \frac{\partial S}{\partial D} \frac{\partial x}{\partial S} \tag{4}$$

The first term requires the sensitivities of the surface NURBS control points with respect the design variables, which could be a challenging task. The second term is exactly the *basis function* of the NURBS, which requires to know the parametric coordinates $\{\xi, \eta\}$ for each surface vertex of the computational grid, but efficient algorithms have been developed [4] for the so called "point inversion problem".

In this formulation, the geometric sensitivities strongly depend on the chosen design variables. The solution is just a mathematical trick consisting in including a new term C that can be easily controlled.

$$\delta x = \frac{\partial C}{\partial D} \left(\frac{\partial x}{\partial C} \right)$$
$$\delta x = \frac{\partial C}{\partial D} \left(\frac{\partial S}{\partial C} \frac{\partial x}{\partial S} \right) \tag{5}$$

The functional chosen for this new term is a Volumetric Uniform Non-Rational B-Spline formulation. We prefer to refer it as a *Control Box*. To understand better how the control box works lets see Fig. 1. There are three distinctive layers: The skin layer, usually generated by the design variables; the outer layer (b), which is required because the control box is a volumetric NURBS, and optional segmentation layers (c) can be set to make the gradient calculation independent from different sections of the geometry, e.g. the upper part of a wing profile from the lower side.

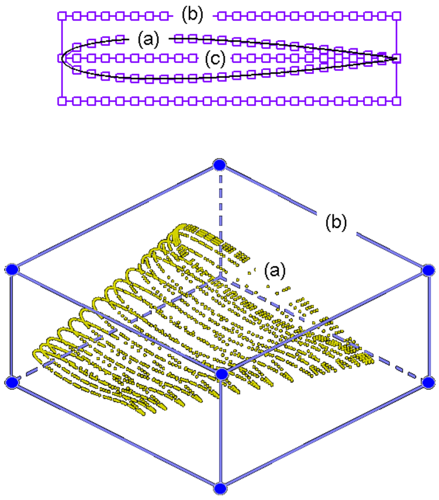

Fig. 1 Concept of the *control box* where three distinctive layers are shown: The skin layer (a), the outer box (b), and the segmentation layer (c).

Gradients are usually calculated on the skin layer from the adjoint solution using the chain rule. From the perspective of the design variables, they only see the control box, effectively hiding all the geometric issues underneath; notice where the brackets are set in (5).

IV. Test Cases

A. Comparison of the geometric sensitivities.

In this test case, the control box is directly employed as design variables. The objective is to compare the geometric sensitivities in three configurations. The baseline is a NACA0012 represented by two NURBS with 30 and 9 control points respectively and the computational grid alone.

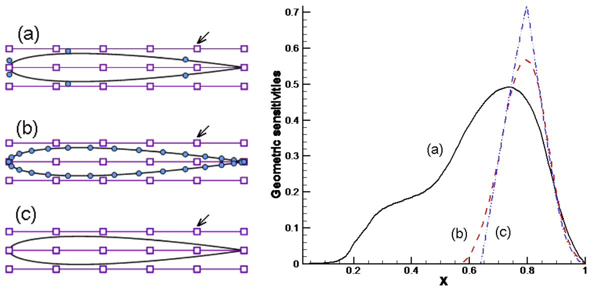

Fig. 2 Comparison of the geometric sensitivities calculated at the control point marked by an arrow in three configurations.

Notice that as the number of surface control points increases, the geometric sensitivities become closer to those that are obtained directly from the computational grid. The main conclusion for this test case is that NURBS are invariant to linear transformations, such as translation and rotation, but it is wrong to perform the same non-linear deformations applied to the computational grid to the NURBS geometric description, for example, Free Form Deformation techniques.

B. Optimization of a NACA0012 airfoil.

The optimization is applied to a NACA0012 airfoil at M=0.8, AoA=1.25º, by employing a bump function in the form:

$$y = y_0 + x(1-x)\sum_k \lambda_k \cdot x^k (1-x)^{1-k} \quad (6)$$

The baseline y_0 is the NACA0012 and k=6. An inviscid optimization is performed on the profile for the same three configurations employed in the previous case: the optimization of two CAD geometries, represented by two NURBS of 30 and 9 control points respectively, and the optimization of the computational grid (CAD-free optimization). The design variables are the polynomial coefficients in (6).

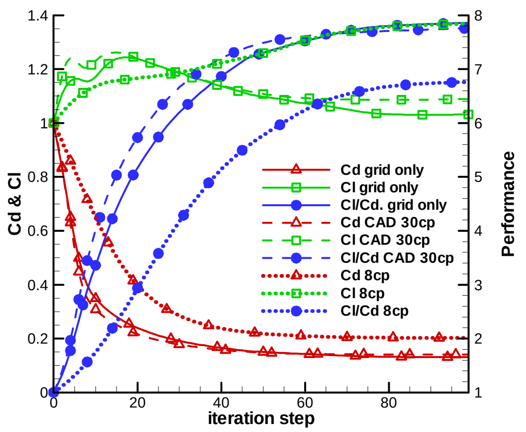

Fig 3. Optimization of a NACA0012 at M=0.8, AoA=1.25º for three representations: CAD-free method, a CAD of 30 control points and a CAD of 9 control points.

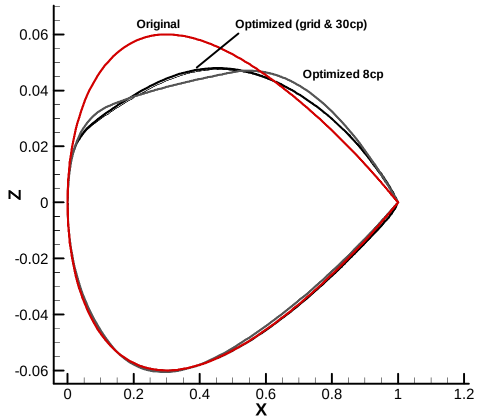

Fig. 4. Optimized profiles from the original NACA0012 at M=0.8, AoA=1.25º for three representations: CAD-free method, a CAD of 30 control points and a CAD of 9 control points.

If the computational grid is a discretization of the geometry, the geometric description, represented by NURBS, is a discretization of the design space, which means that with different geometric representations, different optimal solutions would be obtained.

In this case there is a easy link between the parametric coordinate and the space coordinate, but sometimes this relationship is not as simple to obtain. The final paper will show an optimization employing Sobieczky design variables, and a relevant three-dimensional case.

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