Title: Design of re-entry trajectory optimisation for a SSTO vehicle under uncertainties

This paper addresses the design of the unpowered re-entry of a fully reusable, winged, unmanned single-stage-to-orbit (SSTO) vehicle, as last phase of a payload deployment into low Earth orbit mission. This new generation of space-to-access vehicles represents an alternative to the conventional expendable launch system. By emphasizing full re-usability in their design and employing an airline-like approach, where the cost of acquisition is amortized over repeated flights, these vehicles promise to dramatically reduce the cost per kilogram of access to space. Optimal control strategies can improve the overall safety and reliability of future reusable launch vehicles. In particular, during the re-entry, the control has to steer the vehicle on a feasible trajectory, constrained by acceleration, dynamic pressure and controllability limits. Following the shuttle re-entry approach, two phases are optimised: an atmospheric re-entry phase and a terminal-area energy-management (TAEM) phase. A multi-objective optimisation is performed to minimize the maximum heat flux around the stagnation point, and to minimize the peak heat load. The control laws which optimize the trajectory are the angle of attack and the bank angle. Operational constraints on the dynamic pressure, and maximum axial acceleration and load factor are accounted. Uncertainties are included in the aerodynamic and atmospheric models to evaluate the effects on the vehicle performance. First, nominal optimal control laws are integrated after introducing the uncertainties in the vehicle models, and a statistical analysis on the target states of the final solutions is performed (robustness of the control laws). A second analysis is aimed at determining the best performance of the vehicle when the uncertainties are directly included in the optimization (robustness of the performance).

Trajectory optimization

The adopted hybrid optimization technique is based on a mixed formulation which combines a population-based stochastic algorithm with a deterministic gradient-based method. Population-based stochastic algorithms are able to explore the global search space efficiently, and are able to find a feasible solution when the constraints are sufficiently loose and the number of the design variables is not too high (generally less than 100 variables). Deterministic gradient based solvers can deal efficiently with equality constraints and high dimensionality of the problem. The idea here is to first explore the control search space by using a stochastic approach coupled to a single shooting transcription method to evaluate the performance of candidate solutions, relaxing the system constraints and tolerances on the final states of the system. The second step involves a local optimization which aims to improve the value of the objective function and to ensure that strict equality constraints for the system are satisfied.

Stochastic approach

The optimal control problem is converted into a non linear programming problem (NLP) solved using an hybrid Evolutionary Algorithm (EA) which is obtained by coupling a Multi-Objective Parzen-based Estimation of Distribution[1] (MOPED) and a modified version of the Inflationary Differential Evolution Algorithm[2,3] (IDEA).

The MOPED algorithm belongs to a subset of Evolutionary Algorithms called Estimation of Distribution Algorithms[4] (EDA). These algorithms build a probabilistic model of the search space and the evolutionary search operators, such as crossover and mutation, are replaced by a sampling procedure that operates on the probabilistic model. MOPED is a multi-objective optimization algorithm for continuous problems that uses the Parzen method to build a probabilistic representation of Pareto optimal solutions, with multivariate dependencies among variables. Non-dominated sorting and crowding operators are used to classify promising solutions in the objective space, while new individuals are obtained by sampling from the Parzen model. The Parzen method uses a non-parametric approach to kernel density estimation. For all the individuals within the current population, the Parzen method allocates a probability density function (PDF), each one centered on a different element of the sample. A probabilistic model of the promising search space is built on the basis of the statistical data provided by the individuals, and new individuals are sampled by the probabilistic model itself. MOPED efficiently explores the search space, but often prevents fine convergence on the optimal point, in particular when the solutions are spread over different areas of the feasible space. This feature led to the coupling of MOPED with IDEA, characterized by better convergence properties.

The Inflationary Differential Evolution Algorithm (IDEA) is based on a hybridization of a differential evolution (DE) variant and the logic behind monotonic basin hopping (MBH). The final solutions obtained by MOPED are clustered based on the Euclidean distance between them in the search space, resulting in a variable number of solutions clusters. A DE process is then performed a number of times, beginning with the sub-population of each cluster. Each process is stopped only when the population contracts to below a predefined threshold. Every time the DE stops, a local search is performed in order to converge properly to the local optimum. Since the design optimisation in this case is constrained, the internal DE mechanism can be modified such that the comparison of individuals during the DE process is able to account for the constraints on the system before optimality in terms of the objective function for the system is assessed.

Direct transcription approach

A direct transcription method based on the method of Finite Elements in Time[5] (DFET) on a spectral basis has been implemented for the refinement of the trajectory design. Using this approach, the solution obtained from the previous stochastic optimisations is used as starting point for a gradient based approach, the aim being to further improve the value of the objective function, and better enforce all the constraints on the final states. Compared to the single shooting approach described above, direct transcription methods do not suffer from the same degree of sensitivity to initial conditions and are able to better converge to an optimum point in the presence of rigid constraints on the system. The non-linear programming (NLP) problem is then solved by a gradient-based optimisation method.

Models

The vehicle dynamics assume a point with constant mass is flying around a spherical, rotating Earth. The atmospheric characteristics follow the US Standard Atmosphere 1976 model. The gravitational field is assumed to decrease with the square of the altitude. The lift and drag coefficients were modelled as function of the angle of attack and Mach number, for both hypersonic and supersonic regimes. At hypersonic speeds, the lift coefficient is modelled using the Newtonian theory. The drag coefficient is given by the sum of the induced drag, the engine inlet drag and the drag at zero lift conditions. The heat flux on the vehicle is assumed to be convective, and is a function of the velocity, curvature of the stagnation region and atmospheric conditions.

References

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