Simulation of Unstable Rarefied Gas Flow in Channels and Nozzles

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Abstract. The instability of rarefied gas flows in channels consisting in large deviations of the macroscopic parameters of the flow by very small modification of gas-surface interaction parameters is confirmed in numerical simulations by Monte-Carlo method.

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Our previous analytical and numerical investigations have shown that the limit behavior of the nonlinear dynamic system corresponding to the multiple scattering of rarefied gas particles from the walls of a channel or of a nozzle is not chaotic everywhere [1-3]. It means that the regions of instability are placed only in a subset of the phase space. Numerical results show that chaotic behavior of nonlinear dynamic system corresponds to strange attractors and distinguishes clearly from Maxwellian distribution and from equilibrium on the whole. The main purpose of our new calculations is the investigation of the properties of the nonlinear dynamic system corresponding to a rarefied gas flow. Thus, we are studying the change of the properties of the system with a variation of the scattering function V. Meanwhile we detect the bifurcations of examined type which can be observed in experiments. At once corresponding physical values in the flow are detected. However the problem of the empirical confirmation of the obtained numerically effect is still difficult. The reason for this is that scattering conditions, as well as the regions of the parameters, are quite particular, so they are hardly reproducible experimentally. Considered bifurcations can essentially affect applied in practice different gas flows, such as flows in propulsion systems and in microelectronic vacuum devices. Usually the instability of a system is considered according to Lyapunov definition of the stability: the error between two nearby states of the observed system remain small with time. It means that in an unstable system the solutions with differing insignificantly initial data become appreciably different with time. However we apply another approach based on investigating the behavior of the system by varying the value of some system parameter, which is usually employed in the nonlinear dynamics theory [4] to study Feigenbaum universality in chaos, e.g. for the well-known logistic map. Obtained in our previous analytical and numerical investigations cascade of bifurcations for scattering function V corresponding to ray model [1-3] (it means only one determined velocity of reflected gas atoms, generally different from specular) is simulated numerically by Monte Carlo method for different scattering functions generalizing ray-diffuse reflection of gas particles from the surface. Diffuse addition (multiplied by the coefficient σ) to ray model gives us more general ray-diffuse model, which causes randomization and changes fundamentally the limit behavior of considered dynamic system. The values of the parameters where nonlinear iterative equation describing rarefied gas flow in a long channel becomes unstable (it means sensitivity to boundary conditions) are found from analytical approximations. The main complication in computation is that as the dimension increases the search for a corresponding state requires a lot more computation time and a lot of data to find a suitably close solution (because the amount of required data increases exponentially with embedding dimension). To reduce the computing time our analytical results are applied [1-3]. Flow conditions which are satisfying the requirements to the experiment where the instability of considered type can be detected are indicated.

Typical results are presented by two bifurcation diagrams (fig. 1) and four graphs of the number N of gas atoms along the channel with modified parameters (fig. 2). The bifurcation diagrams are displaying the distribution of gas atoms in the channel for ray-diffuse model of scattering of gas atoms with geometrical parameters of the trajectory of a gas atom in the channel a (x-axis) and $\tan(\theta)$ (y-axis). Near the point of the bifurcation a negligible (less than 1%) change of one of the parameters of the ray-diffuse model causes substantial difference in gas flow in the channel (or in the nozzle).

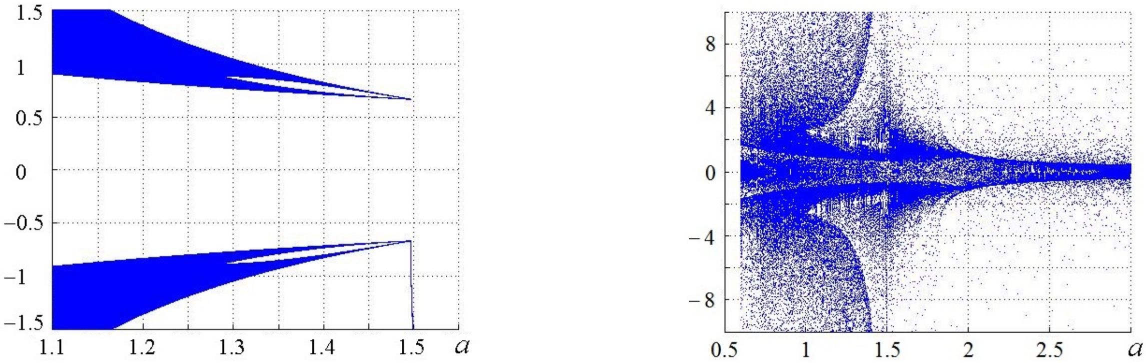

Fig.1: Two bifurcation diagrams, representing the influence of the scattering parameter σ (the share of diffuse scattered gas atoms) on the flow instability for geometrical scattering parameters *a* from 1.1 to 1.55 (a) and from 0.5 to 3 (b), b =1.8 and the initial value $x = \tan(\theta_0)$ from −1.5 to 1.5, $\sigma = 0$ (a) and $\sigma = 0.05$ (b) the ray-diffuse scattering function assumed.

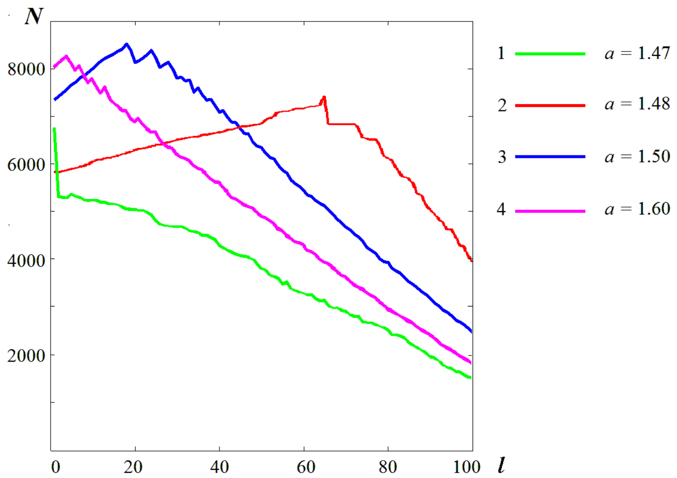

Fig.2: The change of the number N of gas atoms along the channel by the modification of the parameter a = 1.47 (graph 1), a = 1.48 (graph 2), a = 1.50 (graph 3), a = 1.60 (graph 4) by constant b = 1.7, $\sigma = 0.05$, the ray-diffuse scattering function.

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