A Mathematical Formulation of A Novel Adaptive ARE-Based Fault-Tolerant Flight Control System (FTFCS)

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Abstract

We have developed FTFCS for Bell-205 helicopter where the Fault Detection and Accommodation (FDI) has been designed using Artificial Neural Networks (ANN) and real flight-test data [4]. The FDI system has been integrated with an H_∞ controller ([1, 2]). It was found advantageous using μ-Synthesis rather than H_∞ in terms of robustness and performance as shown in [3].

While robust Control theory is centered around the idea of building controllers that cope with model uncertainties, changes that can be accommodated are limited and can not include sensors, actuators, and system components failures. In this paper, we will present a novel approach to extend the robust control theory to designing of an Adaptive FTFCS system which is based on manipulation of Algebraic Riccati Equations (ARE). The concept is applied to sensors failures but it can be, as well, extended to cover actuators and systems components failures following the same line of thought. The system reported here, has two major advantages. The first of which is its **bumpless** transfer capabilities compared to many gain scheduling fault-tolerant schemes. The second feature is **low computational overhead** ([5, 6]). As will be seen, the controller has some fixed components that do not change in case of failures while other scattered components change only whenever a fault occurs.

Control Scheme Formulation under Sensor Fault

The system on which the controller is acting upon is given by:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu}\\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Du}\end{aligned} \tag{1}$$

In the above state space representation, all matrices are assumed constant and, further, $D = 0$. When a sensor fault occurs, the output changes $y \rightarrowtail y_f$ where $y_f = y + \Delta y$ and, thus, the output matrix is perturbed by an amount equals to ΔC. Thus, the new output matrix corresponding to faulty case is given by:

$$C_f = (I + \Delta)C$$

In this treatment, we consider the H_∞ S/KS (sub) optimization design as an illustrative case. The objective to be achieved is:

$$\left\| \begin{matrix} W_1 S \\ W_2 K S \end{matrix} \right\|_\infty < \gamma$$

Where $S = (I + GK)^{-1}$. In order to show the idea more clearly, we assume without loss of generality that $W_1 = I$ and $W_2 = I$ and $\gamma = 1$. If the above plant is connected with the controller , K, , the general interconnected plant is given by:

$$P = \left[\begin{array}{c|cc} \mathrm{A} & \mathrm{B}_1 & \mathrm{B}_2 \\ \hline \mathrm{C}_1 & \mathrm{D}_{11} & \mathrm{D}_{12} \\ \mathrm{C}_2 & \mathrm{D}_{21} & \mathrm{D}_{22} \end{array}\right]$$

where: $B_1 = 0 \qquad B_2 = -B \qquad C_1 = \begin{bmatrix} C \\ 0 \end{bmatrix} \qquad C_2 = C$

$D_{11} = \begin{bmatrix} I \\ 0 \end{bmatrix} \qquad D_{12} = \begin{bmatrix} 0 \\ I \end{bmatrix} \qquad D_{21} = I \qquad D_{22} = 0$

The H_∞ controller is obtained by solving a set of two Algebraic Riccati Equations and those ARE are dependent on the plant matrices and among them the output matrix C which now has in faulty case been replaced by C_f. Definitely, the solution obtained off-line does not hold for the new matrix and, thus, the controller will not be able to control the plant properly. Indeed, in some fault situations, a plant may face severe performance degradation and stability problem if no action is taken on time. Thus, it is desirable to get a new controller that once the fault occurs, it takes over momentarily. As there are various scenarios for the faulty situations, various controller may need to be designed for those situations with some switching mechanism. This requires great amount of knowledge about those faults which needs to be translated into mathematical formulation. Instead, in this approach which is adaptive in nature, it requires solving the new AREs that correspond to the faulty situation. Given the general linear model, the H_∞ controller is obtained by solving the two ARE given by:

$A^T X_\infty + X_\infty A + C_1^T C_1 + X_\infty(\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X_\infty = 0$

$A Y_\infty + Y_\infty A^T + B_1 B_1^T + Y_\infty(\gamma^{-2} C_1^T C_1 - C_2^T C_2) Y_\infty = 0$

In the S/KS case and when $\gamma = 1$, the two AREs are:

$$A^T X_\infty + X_\infty A + C^T C - X_\infty B B^T X_\infty = 0 \qquad \text{(2a)}$$

$$A Y_\infty + Y_\infty A^T - Y_\infty C^T C Y_\infty = 0 \qquad \text{(2b)}$$

With the above AREs, a controller can be formulated. Should a fault occurs, another controller is formulated in view of the fault signature (single or multiple). As such, this approach involves on-line design of controller and switching between current controller and new controller. The controller in the fault-free case

$$K_{norm} = \mathfrak{F}(A, B, C, D, X_\infty, Y_\infty)$$

where X_∞ and Y_∞ are the stabilizing solutions to the two algebraic Riccati equations (AREs), eq- 2, respectively.

On the other hand, in the case of sensor failure, the controller accordingly is given by the folowing theorem

Theorem .1 *When fault occurs, the controller may be expressed as:*

$$K_f = \mathfrak{F}(A, B, C_f, D, \tilde{X}, \tilde{Y})$$

where $C_f = (I + \Delta)C$, and $\tilde{X}$ and $\tilde{Y}$ are the solutions to the following AREs,

$$\begin{aligned} A^T \tilde{X}_\infty + \tilde{X}_\infty A + (C^T + \Delta)^T (C + \Delta) \\ - \tilde{X}_\infty B B^T \tilde{X}_\infty = 0 \end{aligned} \qquad (3)$$

$$\begin{aligned} A\tilde{Y}_\infty + \tilde{Y}_\infty A^T - \tilde{Y}_\infty (C^T + \Delta)^T \\ (C + \Delta)\tilde{Y}_\infty = 0 \end{aligned} \qquad (4)$$

These two new AREs can be considered as slightly perturbed (in the output matrix C) from the original equations. Hence we may reasonably assume that the solutions are closely related by "small" perturbations such as in the forms

$$\tilde{X}_\infty = X_\infty + \Delta_{X_\infty} \qquad (5)$$

$$\tilde{Y}_\infty = Y_\infty + \Delta_{Y_\infty} \qquad (6)$$

Solution Procedure and Implementation

Now let us see how the ARE solutions in such a form can be used in the construction of the controller in the sensor failure situation. Recall that the central controller in the present design problem has the following state space realization (notations in the nominal case),

$$K_{normal} = \left[\begin{array}{c|c} \mathrm{K}_a & \mathrm{K}_b \\ \hline \mathrm{B}^T X_\infty & 0 \end{array}\right] \qquad (7)$$

where

$$K_a = A - BB^T X_\infty - (1-\gamma^{-2})^{-1} Z_\infty Y_\infty C^T C$$

$$K_b = Z_\infty Y_\infty C^T$$

$$Z_\infty = (I - Y_\infty X_\infty)^{-1}.$$

Define

$$\alpha = (1-\gamma^{-2})^{-1/2}$$

and

$$\breve{C} = (1-\gamma^{-2})^{-1/2} C = \alpha C$$

and substitute this into Equation 7 we get,

$$K_{normal} = \alpha^{-1} \left[\begin{array}{c|c} \mathrm{K}_{aa} & \mathrm{K}_{bb} \\ \hline \mathrm{B}^T X_\infty & 0 \end{array}\right] \qquad (8)$$

where

$$K_{aa} = A - BB^T X_\infty - Z_\infty Y_\infty \breve{C}^T \breve{C}$$

$$K_{bb} = Z_\infty Y_\infty \breve{C}^T$$

Such a controller can be decomposed into three cascaded parts, by state similarity transformations and system state space model manipulations,

$$K_{normal} = (I + K_1)^{-1} K_2 (I + K_3)^{-1} \qquad (9)$$

where,

$$K_1 = \left[\begin{array}{c|c} \mathrm{A} & \mathrm{B} \\ \hline \mathrm{B}^T X_\infty & 0 \end{array}\right] \qquad (10)$$

$$K_2 = \left[\begin{array}{c|c} \mathrm{A} & \mathrm{Z}_\infty Y_\infty C^T \\ \hline \mathrm{B}^T X_\infty & 0 \end{array}\right] \qquad (11)$$

$$K_3 = \left[\begin{array}{c|c} \text{A-BB}^T X_\infty & \mathrm{Z}_\infty Y_\infty C^T \\ \hline \mathrm{C} & 0 \end{array}\right] \qquad (12)$$

Similarly, the central controller for the faulty case has the same structure but with:

$$\mathrm{C} \rightarrowtail C_f = (I + \Delta) C$$
$$\mathrm{X}_\infty \rightarrowtail \tilde{X}_\infty = X_\infty + \Delta_{X\infty}$$
$$\mathrm{Y}_\infty \rightarrowtail \tilde{Y}_\infty = Y_\infty + \Delta_{Y\infty}$$
$$\mathrm{Z}_\infty \rightarrowtail \tilde{Z}_\infty = Z_\infty + \Delta_{Z\infty}$$

Accordingly, we have fixed components in the three cascaded controllers and variable components in case failure happens. Those varying components are small and can be used to update the overall controller smoothly.

We have applied this concept to Bell-205 helicopter and the details on the distributed structure of the decomposed controllers will be provided in the full length paper.

References

[1] M. F. Al-Malki. *Fault Tolerant Flight Control System Design with Application to Bell-205 Helicopter, PhD Thesis*. Leicester University, UK, 2004.

[2] M. F. Al-Malki and Da-Wei Gu. Design and Analysis of Helicopter Fault Tolerant Flight Control System -The Lateral Case. *IFAC Symposium on Aerospace Applications, Saint-Petrsburg, Russia*, 2004.

[3] M. F. Al-Malki and Da-Wei Gu. Advantages of using μ-synthesis for fault-tolerant flight control system. *IFAC - China*, 2006.

[4] M. F. Al-Malki and Da-Wei Gu. An ANN-Based Sensors' Fault Detection, Isolation, and Accommodation for Bell-205 Helicopter. Technical Report -Leicester University, UK, August 2001.

[5] P. J. Campo, M. Morari, and C. N. Nett. Multivariable Anti-Windup and Bunipless Tiansfer - A General Theory. *Proceedings of the American Control Conference*, pages 1706–1711, 1989.

[6] Luca Zaccarian and A. Teel. Anti- Windup, Bumpless Transfer and reliable designs: a model based approach. *Proceedings of the American Control Conference*, pages 4902–4907, June 25-27 2001.