

# A Theoretical Prediction of Regression Rates in Swirl Injection Hybrid Rocket Engines

Kohei Ozawa\*, Toru Shimada†

Hybrid propulsion is expected to be applied for various purposes like space transportation, space tourism and space education because of its inherent safety and low cost.

In conventional hybrid rocket engines, liquid oxidizer is injected into a combustion chamber that contains solid fuel, such as HTPB (which is a binder for solid rocket motors). Gasified oxidizer and gasified fuel combust in the boundary layer over the fuel-port wall surface. Hybrid rockets using HTPB have been developed for many years, of which performance dead-ends because the regression rate using HTPB is low (up to 1[mm/s] below the oxygen mass flux of 100[kg/(m<sup>2</sup> · s)] [1]). That is why multi-ports are required hybrid engines using HTPB, and remnants of the fuel in multiport engines decrease launch capability from potential one.

To solve this problem, swirl injection (or vortex injection) method is proposed as a way to realize higher regression rates [2]. This method is to inject liquid oxidizer that has swirl velocity components. The characteristics of this injection method is that the radial pressure gradient caused by swirling makes the flame sheet in the boundary layer more close to the wall of the fuel port. This effect increases the amount of heat transfer from the flame sheet to the wall, and then, higher regression rates can be achieved.

Lab-scale swirl injection hybrid rocket motors have been developed and regression-rate increase by swirling has been studied by several researchers (e.g. Yuasa [2], Knuth [3]). However, there have been few studies which theoretically predict the increase of regression rates by swirling, though they will be useful when designing whatever scale of engines. And then, it will be able to be applied for analysis of various phenomenon such as the combustion stability analysis of swirl engines because theoretical understanding of the effect of each parameter to steady engine performance can help us analyze unsteady combustion property [4].

Based on the above motivation, we are working on the theoretical prediction of the regression rate of swirl-injection hybrid rockets with a swirl flow field model. In this paper the deriving process of the prediction will be explained.

In the present prediction model we assume the followings:

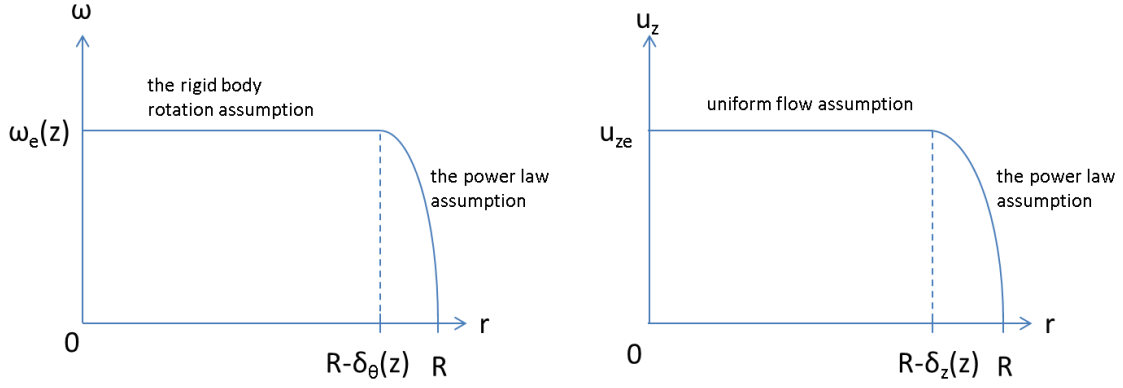
1. The flow in the combustion chamber is axi-symmetric.
2. The axial velocity component is uniform except in the boundary layer over the fuel-port wall surface. (Fig. 1)
3. Tangential velocity components are the same as rigid body rotation except in the boundary layer over the fuel-port wall surface. (Fig. 1)
4. Tangential angular velocity components in the boundary layer obey the power law.
5. The swirl decays exponentially in the axial direction. [5]
6. The influence of blowing from the fuel-port wall surface on the decay constant of swirl is ignored.
7. The axial boundary layer thickness is larger than the tangential one.

About sixth assumption, because it is difficult to estimate the influence of blowing from the fuel-port wall surface on the decay constant of the swirl, as the first step of our study, we will calculate regression rates without taking this effect into consideration.

---

\* Corresponding author, a Master Course Student, Department of Aeronautics and Astronautics Engineering, the University of Tokyo, 3-5-1 Yoshinodai, Chuo-ku, Sagamihara, 252-5210, Japan

† Professor, Institute of Space and Astronautical Science, JAXA, 3-5-1 Yoshinodai, Chuo-ku, Sagamihara, 252-5210, Japan



**Fig. 1 The assumption of tangential angular velocity (Left) and axial velocity distribution (Right) toward radial direction**

Besides the above model, we use the following theories:

1. The Reynolds Analogy
2. The three dimensional mixing length theory [6]
3. The boundary layer theory concerning the blow from the wall [7]

As a result of the present study, we have led the heat flux to the fuel port wall and the regression rates as expressed in the eqs. (1) and (2), respectively:

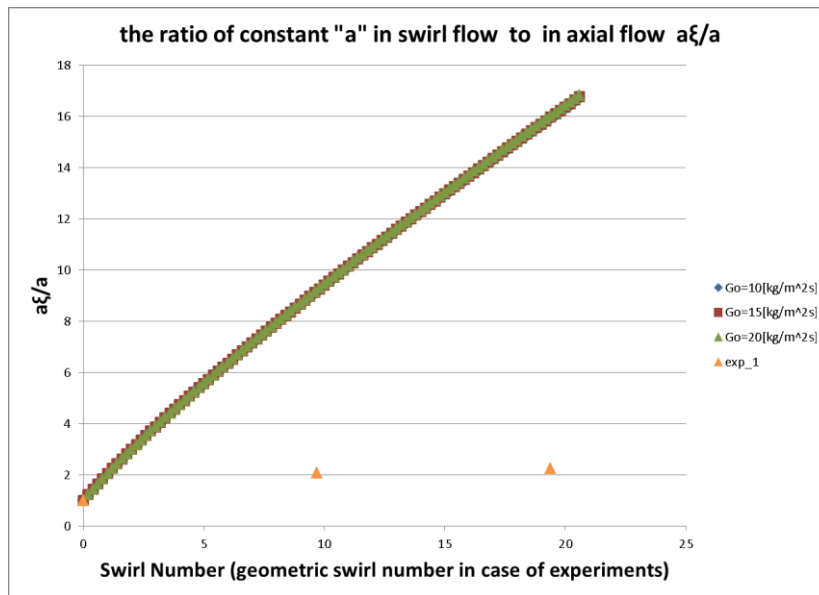
$$\dot{Q}_c = A' \left\{ \frac{1 + (\beta_z/\alpha_z)\xi_0 \exp(pz)}{\left(1 + \frac{\beta_z}{\alpha_z} \xi_0 \frac{\exp(pz)-1}{pz}\right)^{\frac{-\gamma_z}{1-\gamma_z}}} \right\}^{\frac{1}{1-k}} z^{-\frac{0.2}{1-k}} G^{\frac{0.8}{1-k}} \dot{r}^{-\frac{k}{1-k}} \dots (1)$$

$$\frac{a_\xi}{a} = \frac{1 + (\beta_z/\alpha_z)\xi_0 \exp(pz)}{\left(1 + \frac{\beta_z}{\alpha_z} \xi_0 \frac{\exp(pz)-1}{pz}\right)^{\frac{-\gamma_z}{1-\gamma_z}}} \dots (2)$$

Generally, regression rates of the solid fuel are correlated with oxidizer mass flux as the following experimental rule (eq. (3)):

$$\dot{r} = aG_o^n \dots (3)$$

The qualitative properties of these equations are the same as experimental results in the manner that the exponent "n" of the eq. (3) little changes compared with the axial-injection hybrid rocket engine and that the constant "a" changes on a large scale. Next, using these equations, we have compared regression rates with experimental results of a lab-scale swirl injection hybrid rocket engine (Fig.2) [1]. The quantitative properties of the constant "a" are the same order of magnitude, but we cannot say that the present prediction of "a" is accurate.



**Fig. 2** The comparison of the ratio of the constant "a" in swirl flow to one in axial flow (" $a_{\xi}/a$ ")

The first reason of these differences seems that, in eq.1, the influence of the decrease of the tangential friction coefficient is almost ignored. In the axial flow with blowing from the wall surface, axial friction coefficient reduces to 3% to 20% of one in no blowing condition [8]. And then, the amount of the heat transfer to the fuel surface reduces proportional to the friction coefficient [4]. The second reason seems that the actual swirl numbers in the experiments might be smaller than the values calculated from the geometry of experiment instruments. The third cause seems that the approximation that the radius of the pipe is larger enough than the boundary layer thickness, which is used to simplify to lead eq.(1) and (2) easily, may not be suitable in this experiment.

Finally, we will refer to several challenges of this model and an application for it (combustion stability analysis in swirl injection hybrid rockets).

#### References

1. ISAS (JAXA), "the Researching Results Report of Hybrid Rocket Research Working Group", February, 2010
2. Yuasa Saburo, Yamamoto Kengo, Hachiya Hitoshi, Kitagawa Koki and Oowada Youichi, "Development of A Small Sounding Hybrid Rocket With a Swirling-Oxidizer-Type Engine", 37<sup>th</sup> AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, 8-11 July 2001, Salt Lake City, Utah
3. William H. Knuth, Martin J. Chiaverini, J. Arthur Sauer and Daniel J. Gramer, "Solid-Fuel Regression Rate Behavior of Vortex Hybrid Rocket Engines", Journal of Propulsion and Power, Vol. 18, No. 3 (2002), pp. 600-609
4. M. Arif Karabeyoglu, Shane De Zilwa, Brian Cantwell and Greg Ziliac, "Transient Modeling of Hybrid Rocket Low Frequency Instabilities", AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit 20-23 July 2003, Huntsville, Alabama
5. Osami Kito, Hideaki Fujiwara and Koichi Nakabayashi, "the Direct Measurement of the Skin Friction in Swirling Pipe Flow", Transactions of the Japan Society of Mechanical Engineers, Part B, 1985, Vol.51, Num.468, pp. 2597-2605
6. W. Czernuszenko and A.A.Rylov, "A Generalization of Prandtl's Model for 3D Open Channel Flows", Journal of Hydraulic Research, 2000, 38:3, pp. 173-180
7. William H. Dorrance and Frank J. Dore, "The Effect of Mass Transfer on the Compressible Turbulent Boundary-Layer Skin Friction and Heat Transfer", Journal of the Aeronautical Sciences, 1954, Vol.21, pp. 404-410
8. Gerald A. Marxman, "Combustion in The Turbulent Boundary Layer on a Vaporising Surface", Tenth Symposium (International) on Combustion, the Combustion Institute, 1965, pp. 1337-1349