RECEPTIVITY OF FLAT-PLATE BOUNDARY LAYER TO NON-LINEARLY DEVELOPING FREE-STREAM TURBULENCE

M.V. Ustinov
Central Aerohydrodynamic Institute (TsAGI)

**1 Introduction.** The effect of free-stream turbulence (FST) on laminar turbulent transition in a boundary layer has become of great interest during the last decade. General consensus is that boundary layer disturbances in this conditions grow proportionally to Reynolds number based on the boundary layer thickness. It means that transition Reynolds number should be determined by the turbulence intensity only. However the discrepancy in published observations of transition is substantial (see [1]). From this it follows that transition location is not entirely determined by turbulence level, but it is influenced by several factors which are not entirely understood. The most obvious is the influence of length scale of FST. Despite of several studies focused on this factor there is no general agreement among scientists about influence of turbulence scale on transition. Linear theory of boundary layer receptivity developed by Leib, Wundrow & Goldstein [3] states that r.m.s. development of pulsations in boundary layer is described by the universal law

$$\frac{\sqrt{<u'^2>}}{TuR_L} = F\left(\frac{\sqrt{R_x}}{R_L}\right) \qquad (1)$$

where $R_L$ - is Reynolds number based on integral scale of turbulence L.

Results of experiments [1,2] scaled in this way together with theoretical universal amplification curve computed in [3] are presented in Fig.1, *a*. It shows that linear theory [3] strongly underestimates magnitude of pulsations.

Discrepancy between the predictions of linear flat-plate boundary layer receptivity theory and experiment illustrated in previous section may be caused by non-linearity. There are two types of non-linear effects in the FST-induced transition: non-linear evolution of vortical disturbances in outer flow and non-linear development of streaky structures in the boundary layer. Here we shall account the first type of non-linear effect and describe the linear development of disturbances in boundary layer initiated by non-linear turbulence in the outer flow.

**2. Receptivity of boundary layer to single vortical mode.** Let's consider the interaction of grid turbulence with the boundary layer at infinitely thin plate located in the right part of *(x,y)* plane. The oncoming flow has mean velocity $u_\infty$ which is directed along the *x* axis and r.m.s. pulsations $u' = Tu\, u_\infty$, where turbulence level *Tu* is assumed to be small enough. We introduce non-dimensional variables using free-stream velocity and viscose length $l = \nu / u_\infty$ as scales. In these variables all coordinates are equal to the corresponding Reynolds numbers. Vorticity field of FST will be presented as a superposition of periodic in space and time vortical modes. Two types of these modes: streamwise mode $\mathbf{\Omega}_{||}$ with predominantly streamwise vorticity component and cross-flow mode $\mathbf{\Omega}_{\perp}$ with normal to flow direction vorticity will be considered.

$$\mathbf{\Omega}_{||} = a(x)\left\{\mathbf{i}_0 - \frac{\alpha}{\beta}\mathbf{j}_0\right\}e^{i(\mathbf{k},\mathbf{r}-\tilde{\omega}t)}; \quad \mathbf{\Omega}_{\perp} = a_{\perp}(x)\left\{-\frac{\gamma}{\beta}\mathbf{j}_0 + \mathbf{k}_0\right\}e^{i(\mathbf{k},\mathbf{r}-\tilde{\omega}t)} \qquad (2)$$

Here $\mathbf{i}_0, \mathbf{j}_0, \mathbf{k}_0$ are unit vectors along *x,y,z* axes, $\mathbf{k}$ is wavevector of vortical mode and $\alpha, \beta, \gamma$ - are streamwise spanwise and vertical wavenumbers, $\tilde{\omega}$ - is frequency . Further it is assumed that spanwise and vertical periods of vortical modes are large, so cross-flow wavenumbers are small and will be considered as small parameters $\beta \sim \gamma << 1$. Low-frequency disturbances with $\alpha \sim \tilde{\omega} \sim \beta^2$ will be considered further because such perturbations exhibit maximal algebraic growth in the boundary layer [4,5]. Because of flat-plate boundary layer is most receptive to streamwise vortisity, only streamwise modes will be considered further.

In classical linear receptivity theory the interaction between vortical modes is neglected and they correspond to solutions of linearized Navier-Stokes equations. Amplitude of such modes decays exponentially and they are convected with free-stream velocity, so $\tilde{\omega} = \alpha$. In real turbulence vortical disturbances decay more slowly and their phase speed deviates from the free-stream velocity. For this reason we shall consider vortical modes (2) with arbitrary dependence of amplitude from *x* and detuned frequency $\tilde{\omega} = \alpha + \omega$. Such modes can not exist without the interaction with other part of spectrum of FST. The action of other disturbances to the mode will be replaced by the external force ***F***.

Disturbances produced by streamwise mode in the boundary layer $\mathbf{v}(x, y, z, t)$ are governed by Navier-Stokes equations linearized around the basic flow in the boundary layer $\boldsymbol{V_b}$. Due to large streamwise period of perturbations the streamwise pressure gradient can to be neglected and these equations take form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{V}_b, \nabla)\mathbf{v} + (\mathbf{v}, \nabla)\mathbf{V}_b = \nabla_\perp p + \Delta \mathbf{v} + \mathbf{F}; \quad (\nabla, \mathbf{v}) = 0 \qquad (3)$$

Here *p* is perturbation of pressure, $\nabla_\perp$ is gradient in the cross-flow plane. This set of equations is of parabolic type, so initial conditions for *x=0* and boundary conditions at the plate and in the outer flow are necessary. No-slip conditions are set at the plate, initial and outer flow conditions correspond to cross-flow velocity induced by vortical mode (2) in the free stream. For further consideration it is convenient to present the solution of (3) in the following functional form

$$\mathbf{v} = a(0)\left[\mathbf{i}_0 \frac{1}{\beta^2} U + \mathbf{j_0} V + \mathbf{k} W\right] e^{i(\beta y - (\overline{\alpha} + \overline{\omega})T)}; \qquad \{U, V, W\} = \{U, V, W\}(X, \eta, \overline{\alpha}, \overline{\omega}, \Gamma)$$
$$X = \beta^2 X; \quad \eta = z / \sqrt{X}; \quad T = \beta^2 t; \qquad \overline{\alpha} = \alpha / \beta^2; \quad \overline{\omega} = \omega / \beta^2; \quad \Gamma = \gamma / \beta^2 \qquad (4)$$

where normalized velocity components $U, V, W$, coordinates $X, \eta$, wavenumber $\overline{\alpha}$ and frequency $\overline{\omega}$ are values of order of unity.

**3. Receptivity of boundary layer to FST.** Based on solution for single vortical mode, boundary layer velocity pulsations from oncoming turbulence with spectral density of streamwise vorticity $< \omega_x^{\ 2} > (\mathbf{k})$ can be expressed as an integral

$$< u'^2 > = \int \int\limits_{-\infty}^{+\infty} \int \frac{< \omega_z^{\ 2} > (\mathbf{k})}{\beta^4} \left\{ \int\limits_{-\infty}^{+\infty} S(\mathbf{k}, \omega) \left| U\left(\beta^2 x, \eta, \frac{\alpha}{\beta^2}, \frac{\gamma}{\beta}\right)\right|^2 d\omega \right\} \mathbf{dk} \qquad (5)$$

Here $S(\mathbf{k}, \omega)$ is the density of frequency-spectrum of each harmonics of **k-** spectrum of streamwise vorticity in the frame of reference moving with free-stream velocity. For isotropic turbulence and small $\alpha$ spectral density of streamwise vorticity is related to 3d energy spectrum *E(k)* as

$$< \omega_x^{\ 2} > (\alpha, \beta, \gamma) \cong < \omega_x^{\ 2} > (0, \beta, \gamma) = \frac{1}{4\pi} E(\sqrt{\beta^2 + \gamma^2})$$

$$E(k) = Tu^2 L F(k_1); \quad k_1 = kL \quad F(k_1) = \frac{15}{12\pi} \frac{k_1^{\ 4}}{(1 + k_1^{\ 2})^{17/6}} \qquad (6)$$

where $F(k_1)$ is normalized 3d energy spectrum which is approximated by Karman's spectrum.

Frequency spectrum $S(\mathbf{k}, \omega)$ was not ever measured directly or find from DNS of freely decaying turbulence. However it can be related with characteristic correlation time of velocity pulsations in the frame of reference moving with flow velocity coordinates $\tau$

$$S(\mathbf{k}, \omega) = \tau \overline{S}(\omega\tau); \quad \overline{S}(\omega\tau) = \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}(\omega\tau)^2}; \ \tau = \lambda \frac{L}{Tu} G(k_1) \qquad (7)$$

Normalized spectral function $\overline{S}$ responsible for the shape of the time-spectrum is unknown and Gaussian distribution was chosen for its approximation. Expression for the correlation time $\tau$ was found in [6]. Coefficient $\lambda$ in this expression is not determined and will be treated as an empirical constant. Amplitude of vortical mode which is used for computation of function $U$ describing disturbances produced by it in the boundary layer was found from spectral density of streamwise vorticity and time-spectrum as

$$a(\mathbf{k},\omega,x) = \left( < \omega_x^{\,2} > (\mathbf{k},x) S(\mathbf{k},\omega,x) \right)^{1/2}$$

It depends from $x$ indirectly through the dependence from $x$ of turbulence intensity and scale.

Substitution of non-dimensional variables (4) for $\alpha, \beta, \gamma, \omega$ in integral (5) gives the following expression for r.m.s. velocity pulsations in boundary layer

$$\frac{\sqrt{< u'^2 >}}{Tu_0 \sqrt{L_0}} = \Phi(\bar{x}, R_t); \quad \bar{x} = \frac{x}{L_0^{\,2}}; \quad R_t = Tu_0 L_0 \tag{8}$$

where $Tu_0$, $L_0$ are turbulence intensity and scale at the leading edge and $R_t$ is turbulent Reynolds number. Constant $\lambda = 0.2$ was found from the best fit of this solution with experimental data for relatively small distance from leading edge where turbulence intensity and length scale are almost constant.

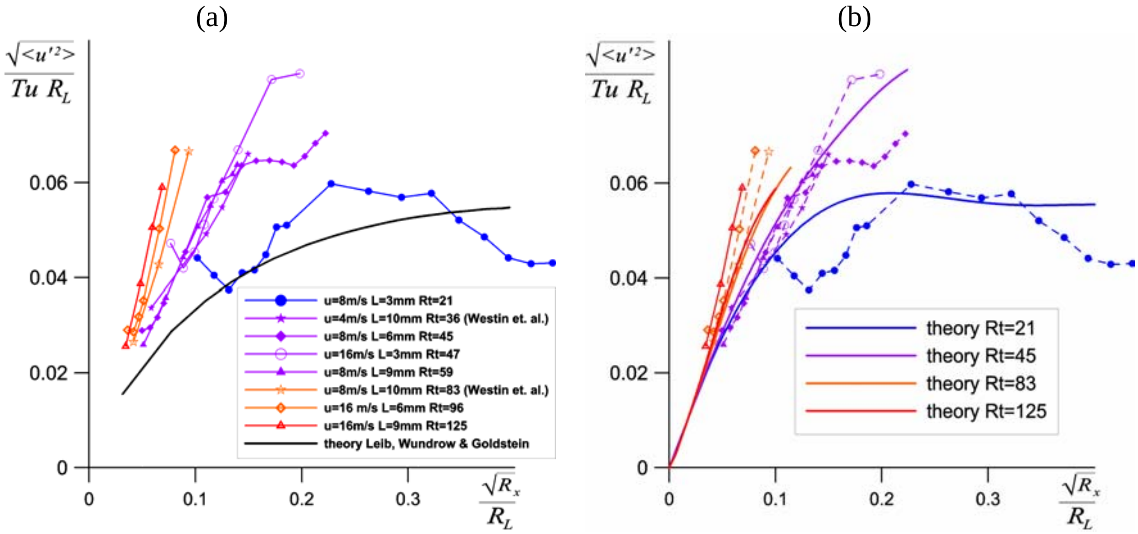


**Fig. 1.** Pulsations in the boundary layer normalized in accordance with (1). Experimental data denoted by symbols. Thick black line – linear theory [3], color solid lines – results of non-linear receptivity theory.

Amplification curves of pulsations in the boundary layer computed for $R_t$=21, 45, 83, and 125 are shown in Fig. 1,b. Coincidence of developed non-linear receptivity theory with experiment is rather good in comparison with linear receptivity theory by Leib, Wundrow & Goldstein [3]. Main advantage of present theory is qualitative description of the enhancement amplification coefficient with the growth of turbulent Reynolds number. However, theory underestimates this trend.

## References


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