Title: Studies on Guidance Law for Interception Problem of Small Velocity Ratio

This material gives a main framework including basic concepts, figures, and some preliminary conclusions.

I. INTRODUCTION

Interception is essentially a rendezvous process of the interceptor and target. According to the relationships of the relative velocity and location, the rendezvous process can be classified as three typical modes: trail interception, head-on impact interception and head pursuit interception. In these modes, the velocity ratio of the interceptor and target is the key parameter related to the successful interception. The trail interception is a widely used traditional interception method due to the fact that the interceptor's velocity is normally greater or even much greater than the conventional target. However, with the increasing of the target flight velocity, typically, such as the reentry flight bodies or the meteorites, the velocity ratio of the interceptor and target became relatively small and even less than 1 in value. In the above conditions, the trail interception method is invalid, and for the other two methods, head-on impact interception and head pursuit interception, further studies are needed to give meaningful conclusions and valid intercept law.

The expected studies in this paper focus on the following aspects:

ⅰ. The single-target-single-interceptor interception problem of small velocity ratio is studied first, the basic guidance laws, including trail interception, head-on impact interception and head pursuit interception, are analyzed with details in both theory and Simulation.

ⅱ. An approximate quadratic optimal guidance law is deduced under a certain simplified conditions. Then, the ideal allowed intercept area of the head pursuit interception is discussed in depth and a special form of this guidance law is given with a straighter intercept trajectory.

ⅲ. To study the cooperative interception problem of small velocity ratio for multi-target, the midcourse guidance law and the terminal guidance law are respectively designed under certain design requirement which generally involves the impact time and angle.

II. BASIC CONCEPTS AND ANALYSIS

This section directly lists the basic concepts and the main results of the analysis.

A. Trail interception

This interception mode is defined as Fig. 1.

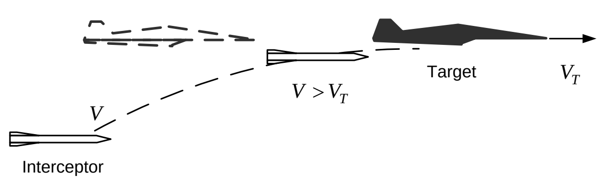

Figure 1. Trail interception.

Analysis results:

1) Relative velocity is effectively decreased due to the chasing characteristics;

2) Launch time limit has been reduced, so the response time of the early warning systems and servo system is extended, conducive to the realization of target acquisition;

3) Rendezvous time is extended because of the small relative velocity, so be the terminal guidance time. This effectively reduces the requirements of the required overload, sensors and computer performance of the interceptor.

B. Head-on impact interception

This interception mode is defined as Fig. 2.

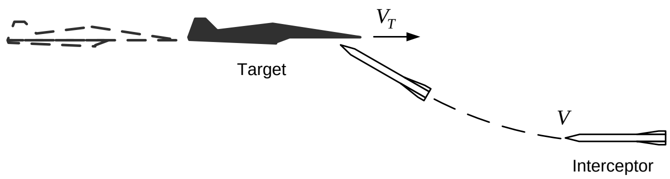

Figure 2. Head-on impact interception

Analysis results:

1) Relative velocity is increased due to the head-on impact characteristics;

2) Response time of the early warning systems and servo system is compressed, requiring that the interceptor enters the designated flight area beforehand;

3) Rendezvous time and terminal guidance time are shortened, so be the. This increases the requirements of the required overload, sensors and computer performance of the interceptor.

C. Head pursuit interception

This interception mode is defined as Fig. 3.

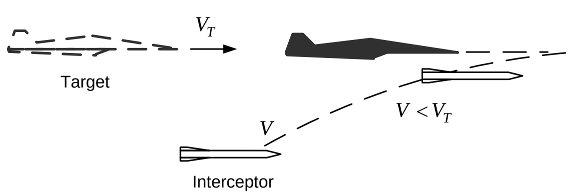

Figure 3. Head pursuit interception

Analysis results:

1) Relative velocity is effectively decreased due to the head pursuit characteristics, so the requirement of the interceptor’s velocity is reduced;

2) Because of the small relative velocity in the terminal guidance, the requirement of the required overload of the interceptor is effectively reduced;

3) This interception method requires that the interceptor accurately enters the front area of target flight path beforehand, so the seeker should be installed in the tail of the interceptor, Bringing additional difficulties in overall design and guidance law design.

III. DESIGN AND ANALYSIS OF THE GUIDANCE LAW

A. State equation of automatic guided system

The movement relationship between the interceptor and target in vertical plane is shown in Fig. 4.

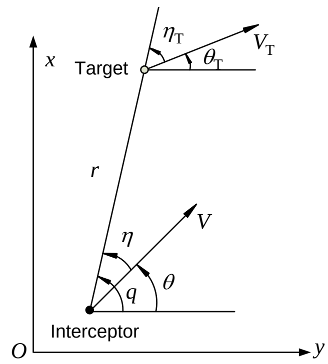

Figure 4. Movement relationship between the interceptor and target

Equation of motion:

$$\begin{cases} \dot{x} = V_T \sin\theta_T - V\sin\theta \\ \dot{y} = V_T \cos\theta_T - V\cos\theta \end{cases}. \quad (1)$$

Simplified expression at the conditions that $\sin\theta \approx \theta$, $\sin\theta_T \approx \theta_T$, $\cos\theta \approx 1$, assign $x_1 = x$, $x_2 = \dot{x}$, the normal acceleration of target is a_T , the normal acceleration of interceptor $a = -u$ (u is defined as the control variable), then the state equation of interceptor is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 + a_T \end{bmatrix} u . \quad (2)$$

B. Design of optimal guidance law based on the quadratic

Quadratic performance index function is selected as the following form

$$\boldsymbol{J} = \frac{1}{2}\boldsymbol{x}^T(t_k)\boldsymbol{\varphi}\boldsymbol{x}(t_k) + \frac{1}{2}\int_{t_0}^{t_k}\left(\boldsymbol{x}^T\boldsymbol{\varphi}\boldsymbol{x} + \boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u}\right)dt . \quad (3)$$

Where, the first part of (3) represents the miss distance requirement $[x_T(t_k) - x_M(t_k)]^2 + [y_T(t_k) - y_M(t_k)]^2$, and this paper simply consider the miss distance along the x direction when $y = 0$, ($\boldsymbol{c} = diag(c_1, c_2)$). The integral term $\boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u}$ represents the control energy, of which $\boldsymbol{R}$ is determined by the interceptor's available overload. The error term $\boldsymbol{x}^T\boldsymbol{\varphi}\boldsymbol{x}$ can be ignored due to the main objective.

According to the optimal control theory at the given interceptor's motion state equation $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u$, the optimal guidance law can be deduced as

$$u = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{x} . \quad (4)$$

Where, $\boldsymbol{P}$ is solved from the Riccati differential equation

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{PA}-\boldsymbol{PBR}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}=\boldsymbol{P}\,,\ \left(\boldsymbol{P}(t_k)=\mathbf{c}\right). \qquad (5)$$

The expression of the optimal guidance law is

$$u=-\frac{(t_k-t)x_1+(t_k-t)^2x_2}{\frac{R}{c_1}+\frac{(t_k-t)^3}{3}}, \qquad (6)$$

or

$$u=-3\left[\frac{x_1}{(t_k-t)^2}+\frac{x_2}{t_k-t}\right],\ c_1\to\infty. \qquad (7)$$

C. Analysis of the intercept aera

Analysis results are shown in the following figures (Fig.5-Fig.7). The shadow area represents the allowed launching area using the above guidance law especially in head pursuit interception. It should be noticed that the intercept trajectory from allowed launching area in Fig.6 may finally switch to the head-on impact interception.

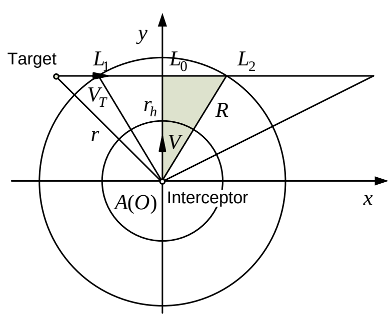

Figure 5. Intercept aera Ⅰ

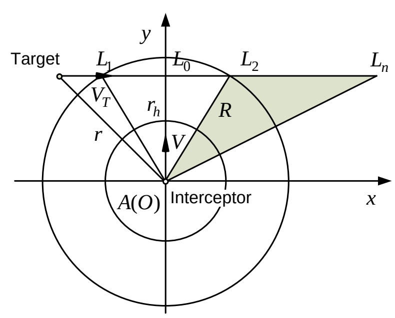

Figure 6. Intercept aera Ⅱ

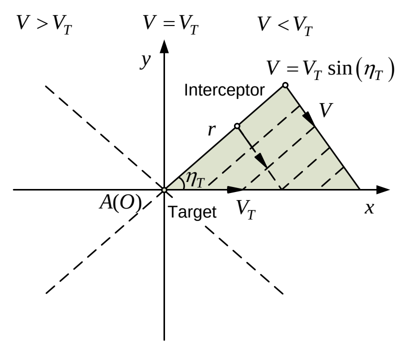

Figure 7. A special form of the guidance law with a straighter intercept trajectory

Fig.7 shows a special form of this guidance law with a straighter intercept trajectory, but also with more stringent requirements of the guidance system because of the necessary accurate measurements of the heading angle η_{T} between the interceptor and target.

IV. COOPERATIVE INTERCEPTION PROBLEM

The long-range interception generally requires two-stage guidance, the midcourse guidance and the terminal guidance. For single-target interception problem, the midcourse guidance needs to give appropriate relative position and velocity for the terminal guidance to guarantee midcourse guidance performance and effectiveness. For the multi-target interception problem, in order to achieve cooperative interception with better interception efficiency, the midcourse guidance and the terminal guidance both need to meet extra requirements compared with that of the single-target interception.

For the midcourse guidance, the cooperative interception not only requires an appropriate relative position and velocity between the interceptors and targets being considered as two integrated single objects respectively, but also an appropriate formation of the interceptors according to the distribution of the targets. Fig.8 is a schematic diagram, as a simplified example of the cooperative midcourse guidance. For the terminal guidance, the cooperative interception needs to meet impact time and angle constraints under the given initial conditions.

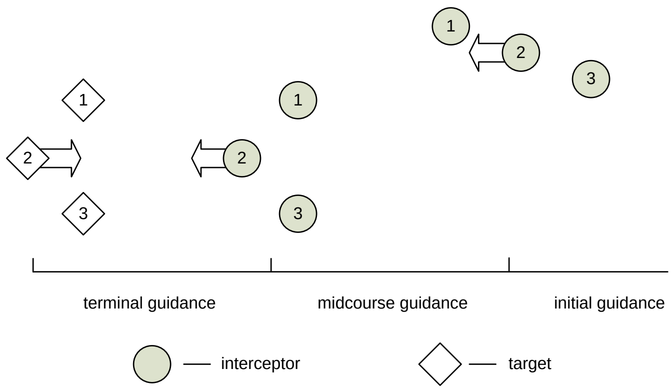

Figure 8. Formation requirement of the multi-target cooperative interception

A cooperative midcourse guidance law for small velocity ratio interception will be given based on the Multi-Model Predictive Control Technology, of which the principle diagram is shown in Fig.9. The state-space model of the relative motion between the interceptor and target is adopted as the prediction model. In this guidance law, the control variable of the state-space model will achieve online correction through predictive control shown in Fig.9 to guide the interceptors to reach the designated locations at a designated time under condition of certain constraints. Also, a cooperative terminal guidance law will be studied which can adjust the impact time and angle to achieve multi-target cooperative interception.

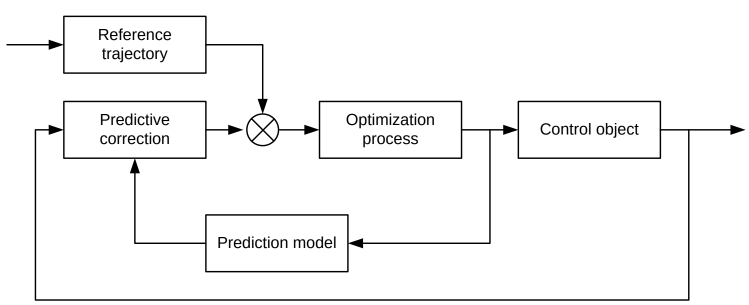

Figure 9. Principle diagram the Multi-Model Predictive Control

The above is the main framework of this paper, and some detailed work remains to be done.