# Application of Gaussian Random Field Model of Rough Surface in Gas Flow Simulation in Channels

*Olga A. Aksenova, Iskander A. Khalidov*
*St.-Petersburg State University, Department of Mathematics and Mechanics,*
198504, Universitetsky pr., 28, Petrodvorets, St.-Petersburg, Russia

**Abstract.** Our new results on the number of crossings of gaussian random field with inclined straight line are applied to direct simulation of rarefied gas flow in a channel with rough walls by Monte-Carlo method.

**Keywords:** Surface roughness, rarefied gas flow, direct simulation, Monte Carlo method.
**PACS:** 43.30.Hw, 68.35.Ct, 83.50.Ha, 83.10.Pp

Velocity distribution and numerical density of rarefied gas atoms in a two-dimensional (plain or axisymmetric) flow in a channel or in a nozzle at high Knudsen numbers is calculated by DSMC method. The main purpose of this paper is to study the dependence of the macro-parameters of rarefied gas molecular flow on surface roughness of the walls [1-6] and on geometrical shape of the channel. Using the expression of scattering function $V$ on rough surface in the form $V = \hat{S} V_0$, where the roughness operator $\hat{S}$ is fully determined by geometrical shape of roughness and by the trajectory of reflected gas atom [3-6], we expand the operator $\hat{S}$ analytically. Expanding the local scattering function $V_0$ in a series in terms of the system of orthogonal functions, we account all the physical and chemical parameters of the gas and of the surface. Unlike more simple models applied by other researchers, such as polygonal-line or conical-hole based models [7-8], our approach allows us to simulate surface roughness on micro-level by a wide class of Gaussian random fields [5]. This statistical model of the roughness permits us to study the roughness operator $\hat{S}$ applying our new results based on the random-field and random-process theory [9-10]. The complexity of the computation of continuum integrals, which must be approximated by the integrals of higher dimension [6], is tackled applying Rice series [5]. Bypassing the most principal difficulty in statistical approach, we reduce the computation time since the most cumbersome and time-consuming step of the computation can be performed prior to the simulation of gas atom trajectories. Such technique of the pre-computation of continuum integrals allows us not only minimizing computation time, but also reducing the number of the parameters. Our algorithm based on the approximation of scattering function $V$ and of the momentum and energy exchange coefficients [1-3] eliminates the complex simulation of the geometrical-shape of the rough surface in the Monte-Carlo method. Additional advantage of considered statistical approach is that not only diffuse-specular model [6], [8], but also Cercignani-Lampis scattering kernel or phenomenological models like proposed in [10] can be applied in the local scattering function $V_0$ approximation. For example, fig.1 shows calculated velocity distribution $P(\theta_1)$ of gas molecules (the directions are defined be the angles $\theta_1$) in different cross-sections of a nozzle with a gorge in the middle section for different roughness parameters $\sigma_1$ (from 0 to 0.5).

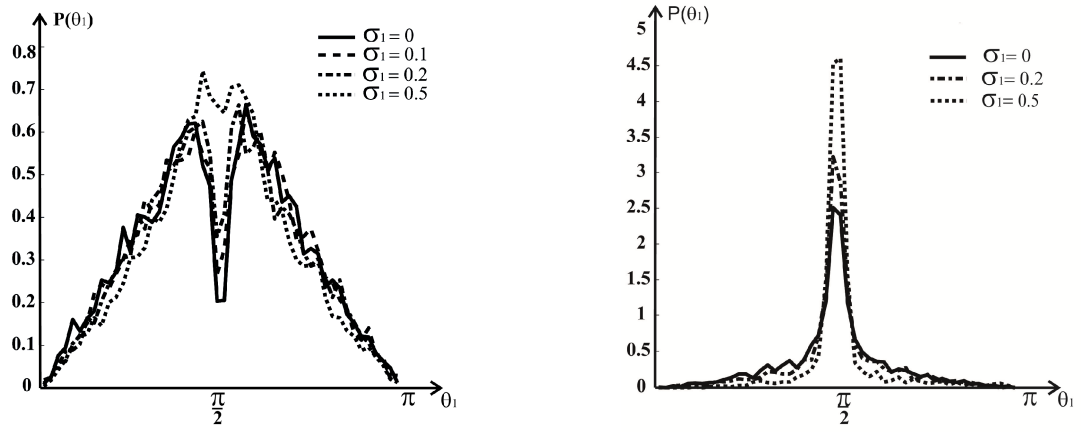


Fig.1: The distribution $P(\theta_1)$ of gas molecules velocities for different values of roughness parameter $\sigma_1$.

## References


[1] Aksenova, O.A., Khalidov, I.A. 2004. Surface Roughness in Rarefied Gas Aerodynamics: Statistical and Fractal Models. St.-Petersburg University Publishers, St.-Petersburg, (in Russian).

[2] Aksenova, O.A., Khalidov, I.A. 2012. In: *Rarefied Gas Dynamics*. American Institute of Physics. AIP Conf. Proc. 1501: Ed. M. Marechal, Melville, New York. pp. 1160 –1167.

[3] Aksenova, O.A., Khalidov, I.A. 2004. *St.-Petersburg University Vestnik*. **1**, No.1. 61–66 (in Russian).

[4] Barantsev, R.G. 1972. Progress in Aerospace Science. **13**. 1–80.

[5] Miroshin, R.N. 1981. Intersections of Curves with Gaussian Processes, St.-Petersburg University Publishers, St.-Petersburg. (in Russian).

[6] Anolik, M.V. , Khabalov, V.D., Khalidov, I.A. 1997. In *Rarefied Gas Dynamics*. Ed. Ching Shen, Peking. 422–427.

[7] Gimelshein, N., Duncan, J., Lilly, T., Gimelshein, S., Ketsdever, A., Wysong, I. 2007. Surface Roughness Effect in Low Reynolds Number Channel Flows. In: *Proc. of XV Int. Symp. on Rarefied Gas Dynamics*. Ed. by Ivanov, M.S., and Rebrov, A.K. Novosibirsk. 695–702.

[8] Ukhov, A., Porodnov, B., Borisov, S. 2009. in *Rarefied Gas Dynamics*. Ed. Takashi Abe. Melville, New York. 712–717.

[9] Aksenova, O.A., Khalidov, I.A. 2012. The Number of Crossings of Gaussian Random Field with a Trajectory of Gas Atom: Application to the Computation of Rarefied Gas Flows. In: *Probability Theory and its Applications*. Int. Conf.: Moscow, June 26-30 (2012), pp. 156 –157 (in Russian).

[10] Aksenova, O.A., Khalidov, I.A. 2012. The Influence of Statistical and Geometrical Parameters on Rarefied Gas Flow in a Channel. *St.-Petersburg University Vestnik*. **1**, No.4. (in Russian).

[11] Yamamoto, K. , Takeuchi, H., Hyakutake, T. 2007. *Phys. Fluids*. **19**, 087102.