

Evolution of turbulent heat flux across a shock wave

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Shock boundary layer interaction (SBLI) is a commonly occurring phenomenon in aerospace vehicles. It occurs in regions such as wing body junction, scramjet inlet and store separation in vehicle. In the region of SBLI, peak pressures, high wall heat flux and boundary layer separation are observed. Numerical simulations of practical geometries having such interactions rely on Reynolds averaged Navier Stokes (RANS) equations. Standard models such as $k - \epsilon$, $k - \omega$, S-A models used in the RANS framework fail to predict the size of separation region, wall heat flux, which are critical parameters in designing the vehicle. This failure is primarily due to some physics which is uncaptured or inadequately modeled in SBLI regions.

One of the main reasons for increased wall heat flux in high speed flows is due to high temperatures near the wall. Therefore, when modeling such flows, it is important to carefully study the physics of unclosed terms in Reynolds averaged energy equation which govern the distribution of mean temperature in the flow. Conventional modeling of turbulent heat flux $\overline{\rho u_i' h'}$ is given by $\frac{\mu_T C_p}{Pr_T} \frac{\partial T}{\partial x_i}$ where μ_T , C_p and Pr_T represent eddy viscosity, specific heat at constant pressure and turbulent Prandtl number respectively. $\frac{\partial T}{\partial x_i}$ represents the temperature gradient along the i axis. For boundary layer flows, a constant value of 0.89 – 0.90 is prescribed for Pr_T . This hypothesis gives satisfactory results when the time scales of mean flow and turbulent time scales are comparable. However, at the shock, mean flow time scales are much smaller than turbulent time scales rendering this hypothesis invalid.

Several attempts have been made to model this term differently. Xiao et al.¹ employed a model that calculates the thermal diffusivity as part of the solution instead of taking a constant value of Pr_T . Bowersox² proposed an algebraic model to compute turbulent heat flux in shock-less zero pressure gradient flows. In-spite of these modeling efforts, the heat flux results do not match with experimental data available in shock dominated flows. The objective of this paper is to study the turbulent heat flux evolution across the shock and propose modeling improvements.

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In order to better understand the evolution of turbulent heat flux across a shock, we take a simplified case of isotropic turbulence interacting with a normal shock. The flow considered is 1-D and steady in the mean, upon which the turbulence is superimposed. This turbulence upon interacting with shock distorts it and the turbulence is in-turn amplified across the shock. This case is free from flow complexities such as flow separation, boundary layer gradient and streamline curvature. There is substantial amount of DNS data^{3;4;5}, theoretical analysis^{4;6} and experiments⁷ carried out on this simplified configuration. These reasons make it ideal for analyzing the flow physics involved in shock turbulence interaction.

A theoretical tool called Linear Inviscid Analysis (LIA) is used to study the underlying process involved in the evolution of turbulent heat flux. LIA models turbulence as linear superposition of waves, each of which interacts with the shock independently thus simplifying the analysis. Results of turbulent statistics across shock obtained from LIA match well with DNS data for the canonical case⁵. In the past, LIA was used to model the effect of shock unsteadiness on the TKE amplification across the shock⁸. Further, this understanding was implemented in RANS framework to solve for SBLI configurations to give improved results pertaining to wall pressure and separation bubble size^{9;10}. In a similar manner, it will be explored how LIA based modeling can improve heat flux predictions.

To study the evolution of turbulent heat flux across a shock, we first take the energy equation written in the frame of reference attached to the shock. Order of magnitude analysis is applied to retain the shock normal derivatives, as they are large compared to derivatives in shock parallel directions. This equation is now transformed back into inertial frame of reference and the higher order terms are dropped out to obtain a linearized governing equation for temperature fluctuations. Taking moment about velocity fluctuations and averaging will give us the equation governing the evolution of $\overline{u'T'}$ across the shock. A corresponding integrated form for $\overline{u'T'}$ can be derived from linearized Rankine-Hugoniot equations, whose budget can be computed using LIA.

The evolution of $\overline{u'T'}$ is first studied for the case of a single 2-D planar wave interacting with the shock. For this simplified case, the upstream pressure, density and temperature fluctuations are considered to be zero. A single vortical wave making an angle ψ_1 with the axis normal to the mean shock, is considered. The property of the wave is defined in terms of complex amplitude, A_v and wavenumber k . Three cases of varying incident angles ($\psi_1 = 45^\circ, 60^\circ$ and 75°) are considered and their evolution is studied for a range of upstream Mach numbers. The budget of the governing equation for each of these cases of ψ_1 will reveal the dominant terms/mechanisms contributing to the evolution of turbulent heat flux.

Based on this understanding of evolution of $\overline{u'T'}$ for a single wave, LIA will be used to compute the evolution of 3D turbulence, which is superposition of multiple waves. Results obtained will be compared with DNS data for verification. The dominant terms will be identified based on the budget in a manner similar to single wave analysis. A suitable closure will be suggested for $\overline{u'T'}$ based on modeling of dominant terms.

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