Stability of high-speed boundary layer on a sharp cone with localized wall heating or cooling

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INTRODUCTION

Prediction of laminar-turbulent transition is important for aerothermal design of high-speed vehicles. This motivates extensive experimental, theoretical and numerical studies of transition at high speeds. Localized surface heating or cooling can be considered as a technique for laminar flow control. This method was used to suppress the first mode disturbances (Tollmien-Schlichting waves) [1-2] by local heating in the subsonic boundary layer. Theoretical and experimental studies showed that stabilization or destabilization of the boundary layer by a heating strip is feasible, depending mainly upon the location of the heating strip.

RESULTS

Numerical studies are carried out for supersonic flow over a cone with sharp leading edge at the free-stream Mach number 6 and the Reynolds number 8x10^6 (based on free-stream parameters and cone length). The fluid is a perfect gas with the specific heat ratio $\gamma = 1.4$ and Prandtl number $\mathrm{Pr} = 0.72$. The viscosity-temperature dependence is approximated by the Sutherland law with the Sutherland constant 110.4 K. The free-stream static temperature is $T_\infty^* = 43.90$ K. The computational domain is a rectangle with its bottom side corresponding to the cone surface. The boundary conditions on the cone surface are the no-slip condition and the constant wall temperature $T_w^* = 290$ K. Asterisk denotes dimensional values.

The problem is solved numerically using in-house implicit second-order finite-volume method described in [3]. Two-dimensional (axisymmetric) Navier-Stokes equations are approximated by shock-capturing scheme that allows for modeling of flow non-uniformities in the temperature jump vicinity. The advection terms are approximated by the third-order WENO scheme to decrease the numerical dissipation. Computational grid consists of 3001x301 nodes. The grid is clustered in the direction normal to the cone surface so that the boundary-layer region contains approximately 50% of nodes.

At first, the steady-state solution, which satisfies the undisturbed conditions, is calculated to provide the mean laminar flow. The three x-locations of heating/cooling strip are considered. Upstream boundary of the strip are $x_h = 0.1, 0.2, 0.3$ respectively, and the strip length is $\Delta x_h = 0.1$ (linear values are made nondimensional by cone length). The wall temperature rise (drop) on the heated (cooled) strip surface is $\Delta T_h^* = \pm 200K$. The heating/cooling strip is modeled by the wall temperature distribution of a 'hat' shape with jumps at the upstream and downstream boundaries of the strip. Figure 1 shows the pressure and temperature fields in the range of $0.15 \le x \le 0.35$ for the heating strip with $x_h = 0.2$.

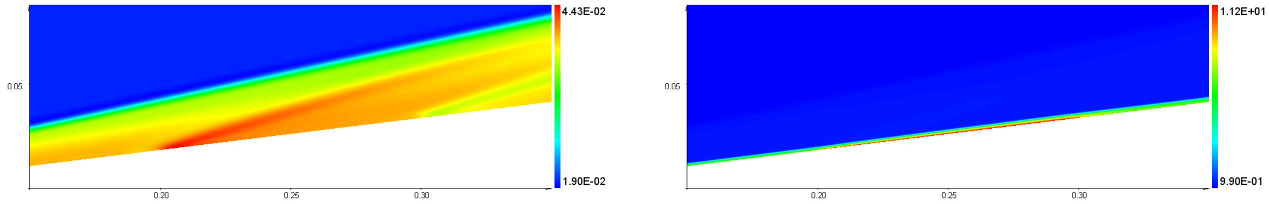

Figure 1. Pressure (left) and temperature (right) fields of the mean flow near the cone surface with heating strip, $x_h = 0.2$, $0.15 \le x \le 0.35$.

The growth rates and downstream amplifications of convectively unstable disturbances are computed with the help of the in-house linear stability code. This code solves the linear stability equations for compressible boundary-layer flow using a 4th-order Runge-Kutta scheme and a Gram-Schmidt orthonormalization procedure. The eigenvalues of the discrete spectrum are calculated with the help of a shooting/Newton-Raphson procedure. Hereafter we focused on 2D disturbances, because the dominant instability is related to the Mack second mode whose maximal growth rates correspond to 2D waves. The transition onset points are estimated using the e^N method. N-factors for the cases of wall heating and cooling with $x_h = 0.1$ are sown in Fig. 2. The heater (or cooler) may cause earlier or later transition depending on the choice of critical N-factor.

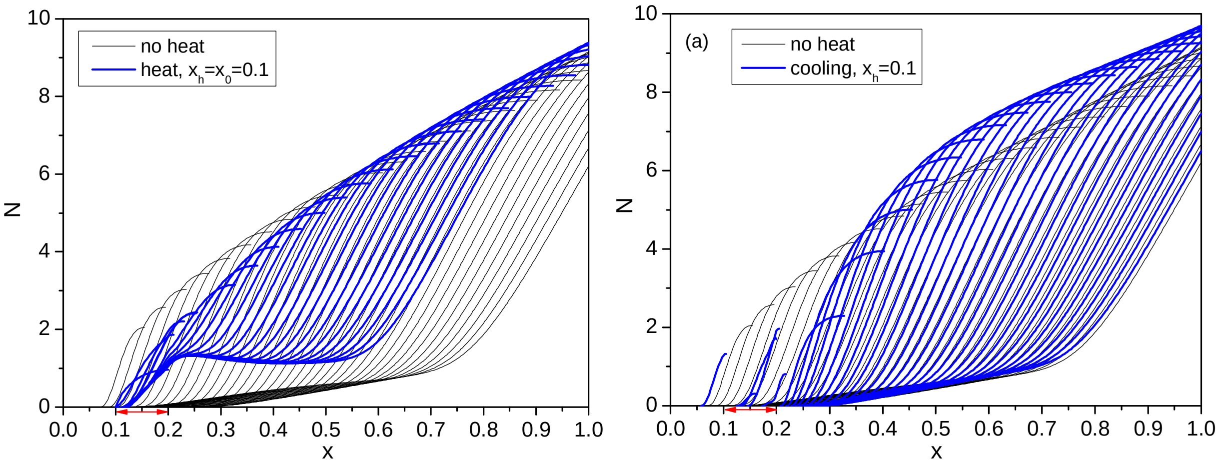

Figure. 2. N-factors for the cases of wall heating and cooling, the red arrows indicate the cooling region.

The e^N method, which is based on the local-parallel stability analysis, may give incorrect results especially in highly non-uniform regions near the heater/cooler boundaries. Therefore direct numerical simulations (DNS) are needed to clarify the local heating and cooling effects.

Two-dimensional axisymmetrical DNS of cone boundary layer stability in the presence of a local wall heating or cooling is carried out. To investigate the boundary-layer instability, initial disturbances are induced into the mean flow by a local periodic suction-blowing in the leading-edge vicinity [3]. The mass flow on the cone surface is given by

$$q_w\left(x,t\right) = \frac{\rho_w^* v_w^*}{\rho_\infty^* U_\infty^*} = \varepsilon \sin\left(2\pi \frac{x-x_1}{x_2-x_1}\right)\sin\left(\omega t\right),\ x_1 \le x \le x_2,\ \ t>0$$

where $\varepsilon = 1\times10^{-4}$ – forcing amplitude was chosen small enough to compare numerical results with LST; $x_1 = 0.05$, $x_2 = 0.064$ – boundaries of the suction-blowing region, ω is the circular frequency. Pressure disturbance field is shown in Fig. 3 for $\omega = 400$ without localized wall heating or cooling. Wall pressure disturbance for this case is shown in Fig. 4.

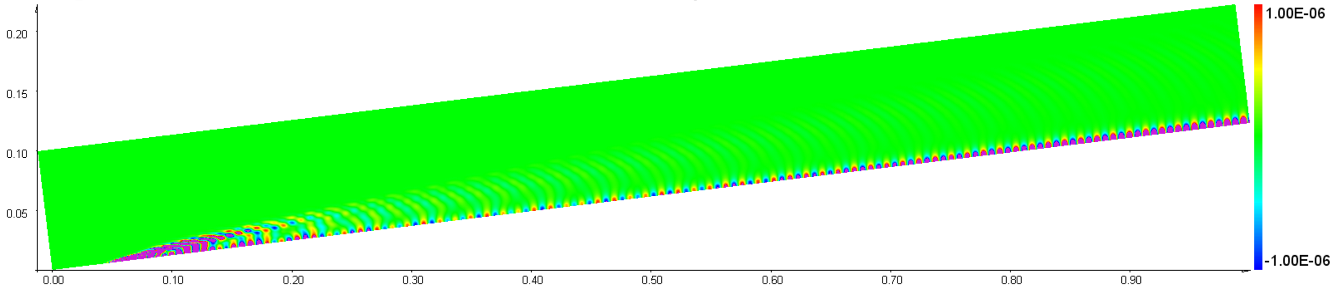

Figure. 3. Pressure disturbance field for $\omega = 400$.

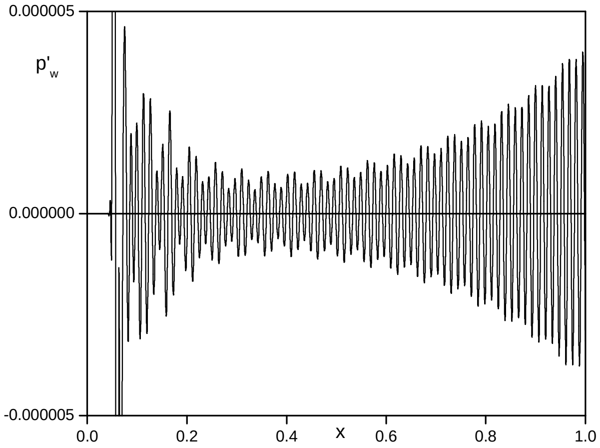

Figure. 4. Wall pressure disturbances, $\omega = 400$ **.**

Results of DNS studies will be compared with LST results for the baseline flow and for the cases with localized heating or cooling.

This work is supported by TransHyBeriAN Project of the 7th Framework Program of the European Commission.

References

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