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A conservative turbulence model for shock-dominated flows

Krishnendu Sinha * and S. J. Balasridhar †

Indian Institute of Technology Bombay, Mumbai, 400076, India.

Interaction of shock wave with turbulent boundary layers is common in many high-speed flows. Examples include deflected control surfaces, supersonic and hypersonic inlet ducts, multi-body aerodynamics and wing-body junctions. Presence of strong shock waves causes boundary layer separation, which can generate additional shocks and expansion waves. Shear layer reattachment downstream of the shock interaction often leads to localized high pressure and heat flux to the vehicle surface. A separation bubble inside an inlet duct acts as a blockage to the flow and can cause unstart. The interaction of turbulent fluctuations in the boundary layer with the shock wave lies at the heart of these phenomena. Shock-turbulence interaction has therefore been the focus of several studies, some of which are discussed below.

Homogeneous isotropic turbulence passing through a normal shock is possibly the most fundamental shock-turbulence interaction. The mean flow is one dimensional and steady, and therefore uniform upstream and downstream of the shock wave. The jump in the mean flow quantities across the shock is governed by the Rankine-Hugoniot relations. Compared to shock-boundary layer interaction (SBLI), the model problem does not have additional complexity due to the flow separation, stream line curvature and boundary layer velocity gradients.

Shock-homogeneous turbulence interaction has been extensively studied using direct numerical simulation.[1−4] This canonical interaction is also amenable to theoretical analysis using rapid distortion theory[5,6] and linear interaction analysis.[7,8] Some limited experimental data is also available in literature.[9] In spite of the geometrical simplicity, the model problem exhibits a range of physical effects, like, generation of acoustic waves, baroclinic torques and unsteady shock oscillations. Physical insight obtained in this canonical problem has proved useful in developing advanced turbulence models for shock-turbulence interaction.[10−12]A sample result, reproduced from Ref. 12, is shown in Fig. 1.

Reynolds-averaged turbulence models can result in large numerical error at flow discontinuities like shock waves. The non-physical behavior of the $k-\epsilon$ solutions are most prominent for strong shock waves, and are probably caused by the non-conservative nature of the source terms in the governing equations. In particular, the source terms contain non-conservative derivatives of the flow variables, and the corresponding discretization error attain large values in a flow discontinuity. Further, the error do not decrease in magnitude with successive grid refinement, as observed for smooth solutions. In some cases, for example, strong shock waves, the error can amplify on fine grids to yield unrealistic values of k and ϵ at the shock.

*Associate Professor, Department of Aerospace Engineering, Senior AIAA Member.

†Graduate student, Department of Aerospace Engineering.

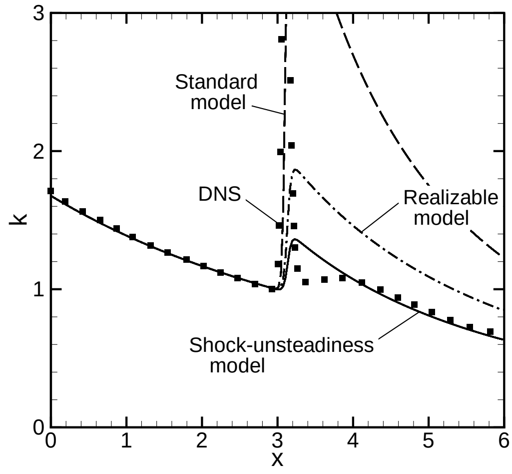

Figure 1: Evolution of turbulent kinetic energy k in the interaction of homogeneous turbulence with a normal shock at Mach 1.5. Different versions of the $k-\epsilon$ model are compared with DNS data.[3]

M	k_0	ϵ_0	DNS source
1.5	2.88×10^{-2}	11.3×10^{-4}	Larsson & Lele[4]
2.0	2.76×10^{-2}	10.7×10^{-4}	
2.5	2.69×10^{-2}	10.4×10^{-4}	Larsson & Lele[4]
2.75	2.66×10^{-2}	10.3×10^{-4}	
3.0	2.64×10^{-2}	10.2×10^{-4}	
3.5	2.61×10^{-2}	10.0×10^{-4}	Larsson & Lele[4]
4.25	2.57×10^{-2}	9.9×10^{-4}	
4.7	2.56×10^{-2}	9.8×10^{-4}	Larsson & Lele[4]

Table 1: Mean and turbulent flow quantities for the interaction of homogeneous turbulence with a normal shock.

In this paper, we systematically study the numerical characteristics of the $k-\epsilon$ solution for canonical shock/turbulence interaction. A finite-volume based CFD code is used for the simulations. The evolution of k and ϵ across the normal shock wave is presented for a range of upstream mean flow Mach numbers (see Table 1). The upstream turbulence quantities correspond to the conditions for which DNS data is presented by Larsson and Lele.[4] Effect of grid refinement on k- and ϵ-amplification at the shock is quantified. Results are also presented for varying upstream values of the turbulence variables.

An alternate conservative form of the $k-\epsilon$ equations are derived and implemented in the finite-volume code. The advantages of the new formulation over the traditional non-conservative $k-\epsilon$ equations is presented for the chosen test cases. The effect of shock strength, grid sensitivity and variability due to changes in inlet conditions are investigated. Finally, future direction towards extending the conservative $k-\epsilon$ formulation to the simulation of complex high-speed flows is discussed.

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