Propagation of disturbances in hypersonic boundary layer on wing with mass transfer through the surface

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To study the propagation of disturbances in three-dimensional boundary layers Wang proposed [1-2] a method for the analysis of the characteristic surfaces. Further this method has been developed in [3-4], where it was used to study two-dimensional flows on a plate and three-dimensional flows on a delta wing. In this paper this approach is used to investigate the spatial boundary-layer flow on a plane wing in the vicinity of the point of inflection of the leading edge. Introduction of cylindrical coordinate system allows parametric investigations of the influence of the sweep angles of the leading edge in a wide range, including forward-swept wings. The flow pattern in a three-dimensional laminar boundary layer interacting with the outer inviscid hypersonic flow past a body of triangular shape is rather complicated even at zero angle of attack and depends considerably on the leading edge sweep and the wing surface temperature. Gas injection through a permeable surface is another important factor affecting the flow. A considerable increase in the injection rate may lead to the boundary layer being pushed away from the body surface with the formation of an inviscid flow region between them [7]. The pressure distribution is affected by the additional, injection-induced displacement thickness, which itself depends on the pressure. Suction through the surface leads to decrease in pressure and boundary layer thickness.

We consider the flat semi-infinite wing with a point of inflection of leading edge in the hypersonic viscous flow at zero angle of attack with mass transfer. The gas is assumed to be perfect with a constant specific heat ratio $\gamma = C_p / C_V$. The flow is considered in the regime of strong viscous-inviscid interaction under the conditions: $M_\infty \to \infty$, $M_\infty \delta >> 1$, where M_∞ is a free-stream Mach number and δ is the dimensionless boundary layer thickness. The temperature of the surface of the wing is assumed to be constant and is given as the temperature factor. In accordance with the hypersonic theory of strong interaction the domain of a flow can be divided into two subregions: the boundary layer and inviscid shock layer. Mass transfer through the surface of the wing is defined by normal velocity V_w, which is order of $V_\infty \delta$ and does not lead to the formation of local inviscid flow near the surface. The distribution of the induced pressure in the shock layer is determined by the "tangent wedge" formula generalized to the nonstationary case [3]. Viscous subregion is studied using the unsteady three-dimensional equations of a laminar boundary layer. The form of the wing is defined by the following parameters: β - the angle between the external flow and the bisector of the angle at the inflection point, Θ - the angle between the bisector and the leading edge. For the numerical solution the dimensionless variables are introduced [5-7], which allows to take into account character of the flow functions in the vicinity of the point of inflection and leading edges. These transformations for the boundary layer equations on a semi-infinite wing allow reducing the problem to a self-similar one [7].

The characteristic surface $f = f(r, \theta, t)$ in the boundary layer, associated with the function $p = p(r, \theta, t)$ of the induced pressure, is a surface on which the derivative $\partial p / \partial f$ is not defined. Condition for the existence of the characteristic surface in the boundary layer and the method of its determination is presented in [3] for a semi-infinite plate and in [4] for spatial flow around a delta wing. In this paper, by introducing the variables, where one of the independent variables is the unknown surface f, the ordinary differential equation is obtained from the system of boundary layer equations. The solution of this equation makes it possible to express the derivative of pressure normal to the characteristic surface. Then the position of the characteristic surface can be determined from the condition of uncertainty of the derivative $\partial p / \partial f$.
We introduce the velocity of disturbance propagation in the following form:

$$a = -\left(\left(\frac{\partial f}{\partial r} \right)^2 + \left(\frac{1}{\Theta r} \frac{\partial f}{\partial r} \right)^2 \right)^{-1/2} \frac{\partial f}{\partial t}.$$

The condition for the characteristic surface is the following integral equation in cylindrical coordinates:

$$\frac{\gamma-1}{2} \int_0^\infty \frac{(H-u^2-w^2)^2}{(a-u\cos(\omega-\Theta_0\theta+\beta)-w\sin(\omega-\Theta_0\theta+\beta))^2} d\eta - \int_0^\infty (H-u^2-w^2)\, d\eta = 0,$$

where H is the total specific enthalpy, u and w are velocity components along the coordinates r and θ respectively, ω is the angle between the direction of disturbance propagation and the external flow, η - coordinate normal to the surface of the wing. This equation allows to determine the average velocity of disturbance propagation, if the velocity and enthalpy profiles are known.

On the basis of self-similar solutions for the boundary layer near the point of inflection of leading edge of a flat wing [6] the propagation of perturbations is studied numerically at different sweep angles of leading edges, surface temperature and various normal velocities for the Prandtl number Pr = 0.72 and the specific heat ratio $\gamma = 1.4$. The results show that the increase of temperature factor from $H_w = 0.1$ to 0.9 increases the rate of propagation upstream in ten times [8]. The relative sizes of the regions of subsonic and supersonic flow in the boundary layer are determined, which allows to explain the observed changes in the upstream velocity of disturbance propagation. Intensive suction through the surface leads to decrease of upstream velocity of disturbance propagation on delta wing ($\Theta = 45°, \beta = 0°$) (Fig. 1). For dimensionless surface temperature $H_w = 0.5$ suction with velocity $V_w \approx -4V_\infty\delta$ “blocks” completely upstream propagation. For $H_w = 0.1$ it occurs at velocity $V_w = -0.2V_\infty\delta$. The influence of suction with $V_w = -V_\infty\delta$ at different parts of the wing with surface temperature $H_w = 0.5$ is shown on fig. 2. Four cases are presented: 1) without mass transfer ($V_w = 0$); 2) suction at the edges of the wing $\theta = \pm 1$; 3) suction at $\theta = -1 \div -0.9$, $\theta = 0.9 \div 1$; 4) suction in central part of the wing at $\theta = -0,5....0,5$. The most influence on the flow is made by mass tranfer near the edges.

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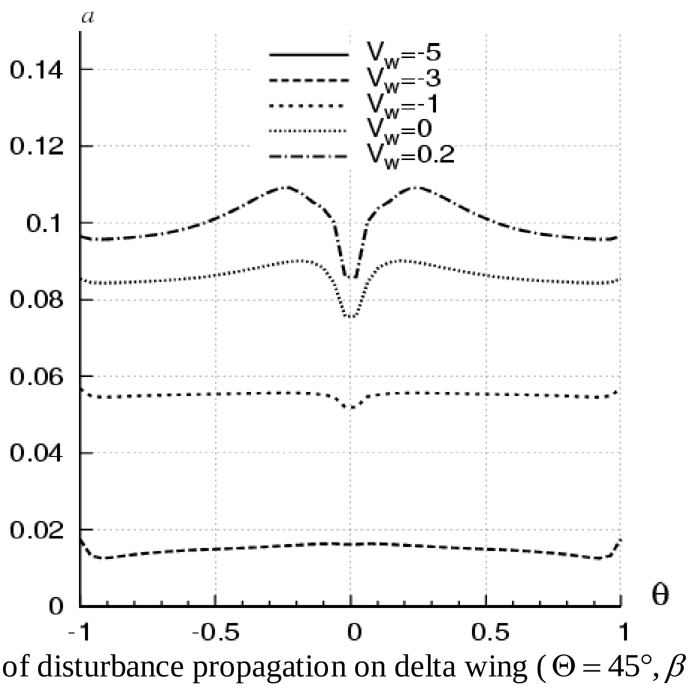

Fig. 1. Upstream velocity of disturbance propagation on delta wing ($\Theta = 45°, \beta = 0°$) with mass transfer.

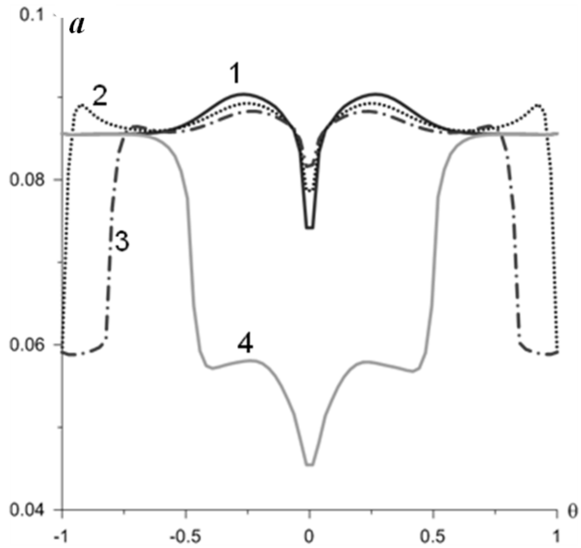

Fig. 2. Upstream velocity of disturbance propagation with suction at different parts of delta wing

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