Determination of the Linear Independence of the Spatial Distribution of Injector Combustion Response in a Subscale Transverse Rocket Combustor

Matthew Wierman and William Anderson
Purdue University, West Lafayette, Indiana, 47907

1.1 Introduction

Combustion instability is the one of the most prominent unsolved problems in rocket propulsion. The coupling of unsteady heat release and chamber acoustics leads to pressure/velocity resonance.(1)(2) This behavior is common in high temperature, high energy density combustion devices like gas turbines and rocket engines. Sufficiently powerful resonance increases heat release to chamber walls and injectors, leading to engine degradation and destruction. Design analysis and proof testing of engines is only possible at fullscale. Past experience with instability has led to complete avoidance of all unstable behavior. NASA standard is that all engines must dampen all instability under instability bombing. (CPIA 655) Present new engine development relies on successful past designs without understanding the reasons for past success. The history of failure in new engine testing and lack of understanding for successful engine programs instill a large risk to innovate.

This paper shall address elements of engineering level model prediction of combustion instability and the interaction between unsteady heat release and gas dynamics that drives the initiation and growth of combustion instability. A subscale combustor is used to provide the means to measure combustion response of the reacting flow from an injector to transverse oscillations at varying levels of amplitude. The study injector can be placed at different positions relative to the transverse mode. Specifically this paper shall address the degree of linear independence of the spatial distribution of velocity-coupled and pressure-coupled injector combustion response. If linearly independence is confirmed, injector combustion response can be interpolated based on injector location in chamber modes and measured behavior at pressure and velocity antinodes. This will simplify models and significantly reduce the number of test cases necessary to construct a combustion response model of a multi-injector combustor.

1.2 Literature Review

1.2.1 Flame Describing Functions (Candel, Lieuwen, Hield)

Nonlinear flame response models have been constructed that address the functional relationship between unsteady heat release and local perturbations of pressure and velocity. (3)(4) Combustion response transfer functions, or Flame Describing Functions (FDFs), have magnitude and phase as a function of local unsteady pressure/velocity. Low amplitude fluctuations have a linear response in magnitude, which concurs with the use of linear input growth rates in previous nonlinear instability models. This approach simplifies computation by taking a solely linear approach. However, high amplitude fluctuations reach a saturation of heat release response magnitude, which is a principal driver of limit cycle behavior. The shift to nonlinear behavior is generally linked to unsteady amplitude, Reynolds number, equivalence ratio, and forcing frequency.(5)

In these cases, combustion response gain and phase are treated as functions of local amplitude and are fitted to an exponential curve to give a rudimentary nonlinear functional form.

$$\frac{q'(t)}{\bar{q}} = n(u_{rms})\frac{u'(t-\tau(u_{rms}))}{\bar{u}}$$

$$n, \tau \sim c + ae^{bu_{rms}}$$

where *n* is the gain, τ is phase lag, *q* is heat release, *u* is velocity, and *a*, *b*, and *c* are empirically determined constants. Terms that are prime and bar represent perturbation and mean quantities, respectively. In Reference 5, *n* and τ were specifically mapped against rms velocity rather than the instantaneous oscillating velocity.(5) The strength of the measured correlation suggests combustion response is a function of amplitude rather than instantaneous unsteady velocity/pressure. Spectral content was greatest at lower frequencies, suggesting that the combustion response acts roughly as a low pass filter. In that case, the combustion would significantly drive only lower frequency modes, and higher frequency modes would arise due to energy transfer between modes.

Used to measure approximate heat release, high-speed imaging of combustion light allows both temporally and spatially resolved measurements of the combustion zone and may lead to improved understanding of the spatial distribution of the combustion as a function of the unsteady amplitude.

1.2.2 Transverse Chamber

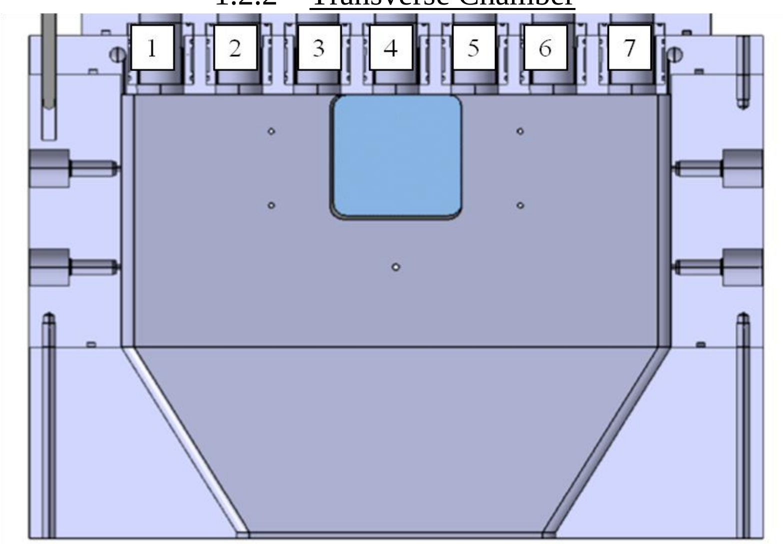

Figure 1- A subscale transverse instability combustor. Elements are ox-centered shear coaxial injectors using JP-8 and decomposed H_2O_2. The chamber width is 10.5 inches. The chamber length including the nozzle is 9.25 inches. The chamber height is 1.5 inches. The window is placed at the center of the chamber and adjacent to the injector face to permit visibility of the flow and combustion light emission of the center element. The center element is located at the velocity antinode of the 1W mode, which is driven by the six outer elements.(6)

The transverse combustor used in this study is a rectangular combustor comprising a linear array of seven ox-centered, fuel swirl elements injecting decomposed H_2O_2 and JP-8 at chamber pressures of ~125 psi to ~ 140 psi depending on the injector configuration.(6)

Unsteady peak-to-peak pressure amplitudes similarly range from 9% to 77% of measured chamber pressure. Measured pressure signals show highly nonlinear, steep-fronted pressure waveforms.

Depending on the injector configuration, the first width (1W) mode ranges from ~1750 Hz to ~2000 Hz with higher harmonics at whole number multiples of the fundamental frequency. Optical accessibility is provided at the center element through a flat quartz window. The observed center element is located at the 1W velocity antinode and a 2W pressure antinode. Its observed response is strongly influenced by both of these effects with a left-right shifting response at 1W and a pulsing downstream mean flow response at 2W.

1.3 Approach

1.3.1 Measure Injector Combustion Response to Unsteady Flowfield

A systematic approach to determination of combustion response is implemented using high sample rate pressure measurements and high-speed movies of combustion light emission that have been analyzed using Proper Orthogonal Decomposition.

$$\begin{aligned}
\ddot{\eta}_n + \omega_n^2\eta_n = -\frac{\bar{a}^2}{E_n^2\bar{p}}\Bigg[& \underbrace{\bar{\rho}k_n^2\int\psi_n(\bar{\boldsymbol{u}}\cdot\boldsymbol{u}')dV}_{[mf_a]} - \underbrace{\bar{\rho}\int\left[\boldsymbol{u}'\times(\nabla\times\bar{\boldsymbol{u}})\right]\cdot\nabla\psi_n dV}_{[mf_b]} + \underbrace{\frac{1}{\bar{a}^2}\int\frac{\partial p'}{\partial t}\bar{\boldsymbol{u}}\cdot\nabla\psi_n dV}_{[mf_c]} \\
& + \underbrace{\frac{\bar{\gamma}-1}{\bar{a}^2}\int\psi_n\frac{\partial p'}{\partial t}\nabla\cdot\bar{\boldsymbol{u}}dV}_{[mf_d]} + \underbrace{\bar{\rho}\int(\boldsymbol{u}'\cdot\nabla\boldsymbol{u}')\cdot\nabla\psi_n dV}_{[nl_a]} - \underbrace{\frac{1}{\bar{a}^2}\int\frac{\partial p'}{\partial t}\boldsymbol{u}'\cdot\nabla\psi_n dV}_{[nl_b]} \\
& + \underbrace{\frac{\bar{\gamma}-1}{\bar{a}^2}\int\psi_n\frac{\partial p'}{\partial t}\nabla\cdot\boldsymbol{u}'dV}_{[nl_c]} + \underbrace{\frac{\bar{\gamma}-1}{\bar{a}^2}\int\psi_n p'\nabla\cdot\frac{\partial\boldsymbol{u}'}{\partial t}dV}_{[nl_d]} + \underbrace{\oint\psi_n\frac{\partial}{\partial t}(\bar{\boldsymbol{u}}\rho'+\bar{\rho}\boldsymbol{u}'+\rho'\boldsymbol{u}')\cdot\boldsymbol{n}dS}_{[bdry]} \\
& - \underbrace{\int\boldsymbol{M}'\cdot\nabla\psi_n dV}_{[mom]} - \underbrace{\frac{1}{\bar{a}^2}\int\psi_n\frac{\partial\boldsymbol{E}'}{\partial t}dV}_{[en]}\Bigg]
\end{aligned}$$

Using the Culick-Galerkin method, this equation identifies the time-dependent pressure amplitude η_n as a second-order harmonic with forcing terms arising from mean flow ($\bar{u}$), unsteady pressure (p') and velocity (u'), modal interactions, boundary layer interactions, and combustion response.(7) The combustion response is contained in the $\frac{\partial E'}{\partial t}$ term as component of a volume integral with modeshape ψ. Mathematically $\frac{\partial E'}{\partial t}$ can be constructed from many linearly independent components, each of which would represent a single injector or a single injector effect. If engineering level models are to effectively use this model of combustion response, pressure and velocity coupled effects must be linearly independent for each injector. The behavior of one injector can be determined based on the behavior of others and position relative to chamber modeshapes.

If linearly independent, injector velocity- and pressure-coupled response can be interpolated based on behavior at pressure and velocity antinodes. Multiple injectors can be mapped without measuring effects at each location. Practically, this would allow velocity- and pressure-coupled response to be determined for a single injector design and mapped across a multiple injector, varying modeshape array.

This is *in lieu* of the time-consuming alternative of measuring the combustion response of each injector position relative to the mode, and underscores the benefit of determining the linear independence of the combustion response.

To produce flame describing functions of each studied injector, measures of local velocity, pressure, and heat release are required. Pressure is recorded using an array of high sample rate pressure transducers. Unsteady velocity is determined from the wave equation using the time-dependent modeshape of the high sample rate pressure measurements.

Combustion light emission is used as an approximation of local heat release. Using OH* and CH* chemiluminescence as measures of heat release is an established technique in combustion analysis.(8)(9)(10) Light emission is recorded using a Vision Research Phantom V7.3 high speed color camera with a light intensifier and a narrow band CH* filter (430 nm). Recorded movies provide large spatially- and temporally resolved data sets that need to be carefully processed to provide meaningful input to the construction of flame describing functions. For this analysis, proper orthogonal decomposition is used. Proper orthogonal decomposition (POD) is an established method to obtain low-dimensional approximations of high-dimensional phenomena.(11) Useful image sizes and time scales can be processed in their entirety using singular value decomposition on a desktop computer.

1.3.2 Transverse Combustor

Using the Transverse Chamber at select unsteady amplitudes, the combustion response of injectors at different modeshape positions are measured and used to determine and test the linear independent combustion response of velocity and pressure coupling.

Past testing with this combustor has successfully demonstrated various amplitudes of spontaneous, repeatable combustion instability and successful performance of optically accessible windows for recording combustion light emission.(6) Combustion light emission is recorded at a variety of modeshape positions with respect to pressure and velocity nodes and antinodes. To test linear independence of the injector combustion response, the combustion field of each studied injector needs to be optically accessible.

In addition to combustion response with respect to modeshape, the effects of changes in injector configuration are studied, specifically different injector lengths which are known to affect stability behavior.(12) This allows testing of identical injector flowrates and chamber pressure at several discrete ox post resonances. Pressure measurements are used to create a map of modeshape for all test cases. Measured chamber modeshapes shall be used to interpolate spatial scaling of local pressure/velocity effects. This map shall be interpolated for each injector position for a measure of local pressure and velocity at each mode.

General combustion response and flame describing functions are determined for pressure and velocity effects at each injector by converting light emission temporal component and unsteady pressure/velocity to frequency space and decoupling gain and phase from the resulting flame describing function. Using the measured pressure and velocity modeshapes, linear separation of effects will isolate pressure-dependent and velocity-dependent response for each of the three studied injectors.

A proposed form of the linear independent combination of the gain of the combustion response is

$$n = n_1\psi + n_2\nabla\psi$$

$$\frac{q'(t)}{\overline{q}} = n_1\frac{p'(t-\tau)}{\overline{p}}, n_2\frac{u'(t-\tau)}{\overline{u}}$$

where n_1 and n_2 are the respective pressure-coupled and velocity-coupled responses and ψ is the pressure modeshape. Through solving this as a system of equations for observed injectors, independent combustion responses will be determined for unsteady velocity and pressure and used to predict other injector combustion responses through interpolation on measured modeshapes.

1.4 Example Results

Based on measurements of combustion response analyzed by POD for each studied injector, flame describing functions in gain and phase are generated as functions of local normalized unsteady pressure and velocity amplitude. These can be represented as individual 2-D traces as functions of amplitude and as 3-D surfaces as functions of amplitude and frequency or position (Figure 2).

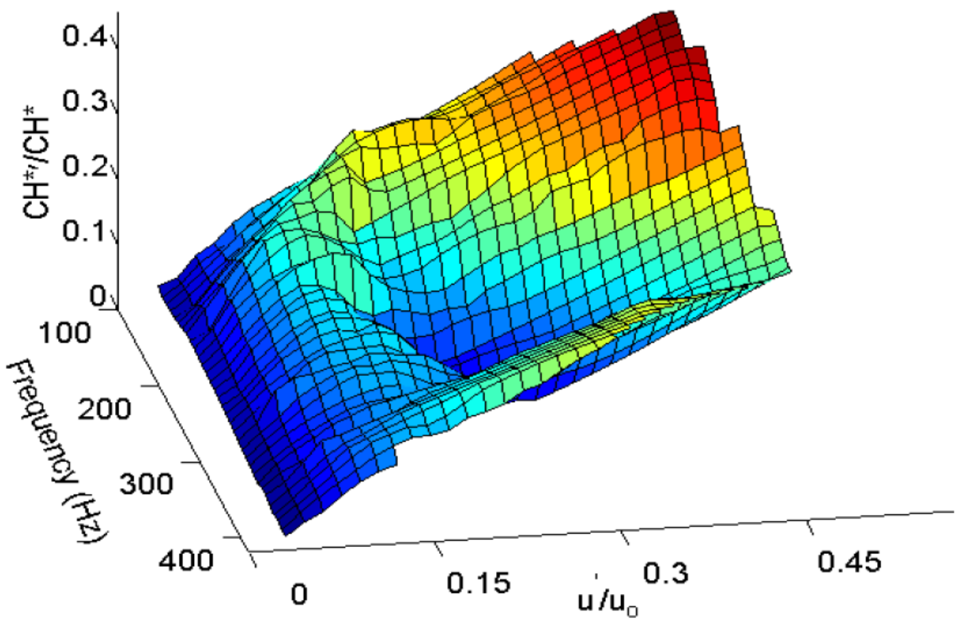

Figure 2 - Example gain of flame describing function as a function of normalized unsteady velocity and frequency. Data from the proposed study shall be represented in a similar fashion for each injector as functions of frequency and local unsteady normalized velocity and pressure.(13)

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