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DYNAMICS OF MARINE STATIONARY PLATFORM UNDER ACTION OF HORIZONTAL SEISMIC LOADING

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Abstract. *Dynamics of a marine stationary platform under action of the seismic loading is investigated. The structure consists of a tube-like ferroconcrete rod of variable cross-section. To describe the rod vibrations the beam-like model is used. The system of partial differential equation is reduced to the system with 6 degrees of freedom. Some numerical examples are studied. It is established that at design it is necessary to pay the main attention to the concrete strength when the extension deformations appear.*

INTRODUCTION

One of the possible structures of a marine stationary platform for the oilindustrial engineering is studied. The platform consists of a tube-like ferroconcrete rod with the upper operating floor. This structure may be used for the depth of see near 250 m. At the design of structure some external forces are to be taken into account. Among them there are the influence of a surface gravity waves, of a wind acting on the operating floor, of a water stream, and at last of an earthquake. For this structure the influence of surface waves is investigated in [1]. Here we study only the action of a horizontal seismic excitation and find the rod bending deformations. Peculiarity of this system is that its lowest natural frequency is much smaller than the typical frequency of the seismic excitation. That is why we seek the solution as a sum of 6 natural modes. For calculation we take not real, but model seismic impulse which alloys us to find the dependence of rod vibrations on its frequency. The main result of this investigation is that at design it is necessary to pay the main attention to the concrete strength when the extension deformations appear.

1 MATHEMATICAL MODEL OF PLATFORM

The structure consists of a tube-like ferroconcrete rod of variable cross-section. Below the rod is attached to the foundation which can move in the horizontal direction and rotate around the horizontal axis. The operating floor is attached above the rod. Vibrations in one plane ate studied. The attached mass of water and the resistant force of water proportional to the square of the cross-section velocity are taken into account. It is assumed that the water is stationary, influence of the surface waves and of the stream is ignored. The seismic load is horizontal, and the ground acceleration $a(t)$ is given as a function of time and in the limits of foundation it is not depend on the spatial co-ordinates. The mathematical model used here is described in [1], the difference is that instead the surface waves here the action of the seismic load is studied.

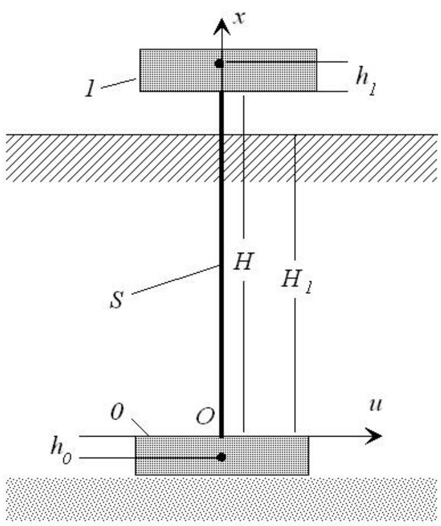

Figure 1: The structure diagram.

The foundation and the basic platform are supposed to be rigid bodies *0* and *1* (see Fig. 1), which are attached to the rod S. For the rod the beam model is accepted. Equation of the rod bending in the movable co-ordinate system is

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial}{\partial x}\left(P(x)\frac{\partial u}{\partial x}\right) + \rho(x)\frac{\partial^2 u}{\partial t^2} = f(u,x,t), \tag{1.1}$$

where

$$M = J(x)\left(\kappa\psi(\kappa,x) + \frac{\beta_0}{\omega}\frac{\partial\kappa}{\partial t}\right), \quad \kappa = \frac{\partial^2 u}{\partial x^2}, \quad f = -\rho(x)a(t) - f_v. \tag{1.2}$$

Here $u(x,t)$ is the horizontal deflection of the rod cross-section x, $0 \le x \le H$, H is the rod height, M is the bending moment, κ is the curvature, J is the rod bending stiffness in linear approximation, f is the external load her unit length. The moment M is given with the account of the nonlinear visco-elastic properties of the cross-section materials. The function ψ is describer in Section 2, in linear approximation $\psi = 1$. In (1.2) β_0 is the dimensionless resistant coefficient, ω is the typical frequency.

The water resistance force per unit length is denoted as f_v and it is equal to

$$f_v = C_v\gamma_w R(x)v|v|, \quad v = \frac{\partial u}{\partial t} + v_0, \quad v_0(t) = \int_0^t a(t)\,dt, \tag{1.3}$$

where C_v is the dimensionless coefficient, γ_w is the water density, $R(x)$ is the cross-section diameter.

In (1.1) $P(x)$ is the axial compressing force

$$P(x) = \left(m_1 + m_s - \int_0^x \rho_s(x)\,dx\right)g, \quad m_s = \int_0^H \rho_s(x)\,dx, \tag{1.4}$$

where m_1 and m_s are the masses of the body *1* and of the rod, $\rho_s(x)$ is the rod density per unit length, $g = 9.81\,(m\,c^{-2})$ is the gravity acceleration

$$\rho(x) = \rho_s(x) + \xi(x)\rho_w(x), \quad \rho_w(x) = \pi R^2(x)\gamma_w, \tag{1.5}$$

$\rho_w(x)$ is the density of the additional mass of water, and $\xi(x) = 1$ at $x < H_1$, $\xi(x) = 0$ at $x > H_1$, $H_1 < H$ is the depth of water.

The boundary conditions at the bottom $x = 0$ and at the upper end $x = H$ are the motion equations of the bodies *0* and *1*

$$\begin{aligned}
&m_0\left(\frac{d^2u_0}{dt^2} + a\right) = -\left.\frac{dM}{dx}\right|_{x=0} - P(0)\varphi(0) + F_0, \quad u_0 = u(0) - h_0\varphi(0),\\
&J_0\frac{d^2\varphi(0)}{dt^2} = M(0,t) - h_0\left.\frac{dM}{dx}\right|_{x=0} + L_0 + (h_0 - h_*)F_0,\\
&m_1\left(\frac{d^2u_1}{dt^2} + a\right) = \left.\frac{dM}{dx}\right|_{x=H} + P(H)\varphi(H), \quad u_1 = u(H) + h_1\varphi(H),\\
&J_1\frac{d^2\varphi(H)}{dt^2} = -M(H) + h_1\left.\frac{dM}{dx}\right|_{x=H}, \qquad \varphi = \frac{\partial u}{\partial x},
\end{aligned} \tag{1.6}$$

where $m_0,\ m_1$ are the bodies masses, $J_0,\ J_1$ are the central inertia moments, $h_0,\ h_1$ are the distances to the mass centers (see Fig. 1) of the bodies *0* and *1* respectively.

The non-linear force F_0 and moment L_0 of the ground and the foundation interaction are accepted as [2]

$$F_0 = -c_u u_* \phi_u(u_*) - \frac{\beta_u c_u}{\omega}\dot{u}_*, \quad u_* = u(0) - h_*\varphi(0),$$
$$L_0 = -c_\varphi \varphi_0 \phi_\varphi(\varphi(0)) - \frac{\beta_\varphi c_\varphi}{\omega}\dot{\varphi}(0), \tag{1.7}$$
$$\phi_u = \left(1 + \left|\frac{u}{u_{cr}}\right|^{m_u}\right)^{-1/m_u}, \qquad \phi_\varphi = \left(1 + \left|\frac{\varphi}{\varphi_{cr}}\right|^{m_\varphi}\right)^{-1/m_\varphi},$$

where h_* is the distance to the force horizontal center of the ground and the foundation interaction, parameters $c_u, c_\varphi, \beta_u, \beta_\varphi, m_u, m_\varphi, u_{cr}, \varphi_{cr}$ are to be given. In linear approximation $\phi_u = \phi_\varphi = 1$. The second summands in relations (1.7) take into account the visco-elastic ground properties.

2 THE BENDING ROD STIFFNESS

The rod is the three layered ferroconcrete tube with the variable cross-section (see Fig. 2). The outer layers $r_1 \le r \le r_2$ $r_3 \le r \le r_4 = R$ are steel, and the middle layer $r_2 \le r \le r_3$ is concrete.

$$r_k(x) = r_{k0} - bx, \; k = 1,2,3,4, \quad b = \frac{R_0 - R_1}{H}, \tag{2.1}$$

where r_{k0} are the layers radii at $x = 0$, R_0 and R_1 are the radii of lower and upper tube cross-sections.

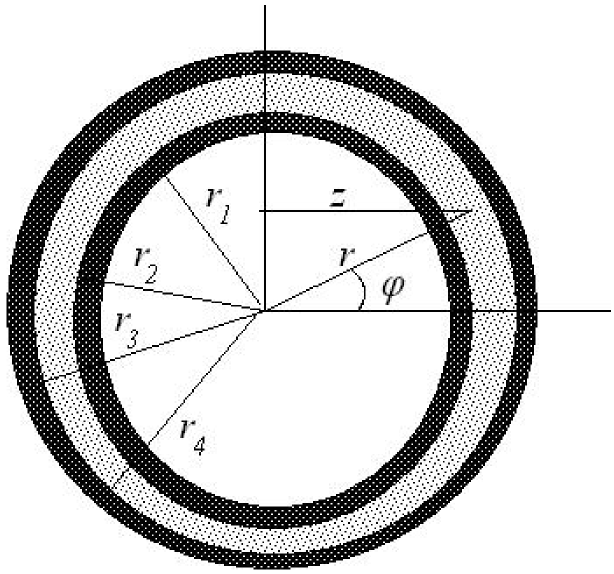

Figure 2: The three layered tube cross-section.

We accept the following dependences between stresses σ and strains ε (in the vertical direction neglecting the other stresses and strains):
for the steel

$$\sigma = E_s \begin{cases} -\varepsilon_s & at \quad \varepsilon < -\varepsilon_s, \\ \varepsilon & at \quad |\varepsilon| \le \varepsilon_s, \\ \varepsilon_s & at \quad \varepsilon > \varepsilon_s, \end{cases} \qquad \sigma_s = E_s \varepsilon_s \tag{2.2}$$

and for the concrete

$$\sigma = E_c \begin{cases} -\varepsilon_c & at \quad \varepsilon < -\varepsilon_c, \\ \varepsilon & at \quad -\varepsilon_c \le \varepsilon \le 0, \\ 0 & at \quad \varepsilon > 0, \end{cases} \qquad \sigma_c = E_c \varepsilon_c. \tag{2.3}$$

Here E_s E_c are the Young modules for steel and for concrete, ε_s is the yield limit of steel which is accepted identical at extension and at compression, ε_c is the flowing limit for concrete at compression. Relations (2.2) and (2.3) correspond to the Prandtl model, and relation (2.3) fixes the assumption that the concrete does not resist the extension. In the linear approximation the both materials are linearly elastic ones, and for the steel $\sigma = E_s\varepsilon$ and for the concrete $\sigma = E_c\varepsilon$.

The rod cross-section x is compressed by the axial force P and let the curvature $\kappa(x)$ is given. Our aim is to find the dependence between the bending moment M and the curvature κ. It is supposed that the hypothesis about the plane cross-sections if fulfilled. According to this hypothesis the deformation ε of the fibre lying at the distance z from the cross-section diameter is equal

$$\varepsilon(r, \varphi) = \varepsilon_0 + \kappa z, \quad z = r\cos\varphi. \tag{2.4}$$

In the linear approximation

$$\begin{aligned} &P = -K\varepsilon_0, \qquad K = \pi\left(E_s(r_2^2 - r_1^2) + E_c(r_3^2 - r_2^2) + E_s(r_4^2 - r_3^2)\right), \\ &M = J\kappa, \qquad J = \frac{\pi}{4}\left(E_s(r_2^4 - r_1^4) + E_c(r_3^4 - r_2^4) + E_s(r_4^4 - r_3^4)\right). \end{aligned} \tag{2.5}$$

Due to the compression $P > 0$ we get $\varepsilon_0 < 0$, therefore for the small enough curvature κ the linear relations (2.5) are valid. At

$$|\kappa| > \kappa_* = \min\left\{\frac{\varepsilon_0 + \varepsilon_s}{r_4}, \frac{\varepsilon_0 + \varepsilon_c}{r_3}, \frac{-\varepsilon_0}{r_3}\right\} \tag{2.6}$$

one or the both materials become as the nonlinear ones and the relations (2.5) are to be corrected.

$$P = \int \sigma(r, \varphi, \varepsilon_0)\, r\, dr\, d\varphi, \qquad M = \int \sigma(r, \varphi, \varepsilon_0)\, r^2 \cos\varphi\, dr\, d\varphi, \tag{2.7}$$

where the integration is fulfilled on the circular area occupied by materials, and the stress $\sigma(r, \varphi, \varepsilon_0)$ is calculated by the relation (2.2) or (2.3). If the curvature κ and the compressing force P are given then by the first relation (2.7) we find the deformation ε_0, and then by the second relation (2.7) we calculate the moment M. Therefore the function $\psi(\kappa) = M/(J\kappa)$ is build.

3 THE MODEL OF THE GROUND ACCELERATION AT THE EARTHQUAKE

For the analytical description of the acceleration $a(t)$ we use the relation

$$a(t) = At^2 e^{-\alpha t}\sin(\nu t + \theta). \tag{3.1}$$

It is necessary to put the following restriction on the function $a(t)$

$$\int_0^\infty a(t)dt = 0, \tag{3.2}$$

because in the opposite case the ground will move after the seismic impulse is finished. For the function (3.1) the relation (3.2) is fulfilled if

$$\alpha(3\nu^2 - \alpha^2)\sin\theta + \nu(\nu^2 - 3\alpha^2)cos\theta = 0. \tag{3.3}$$

In Fig. 3 the impulse with the parameters $A = 0.295$, $\alpha = 0.4$, $\nu = 10$ is shown (the value θ is to be found from the equation (3.3)).

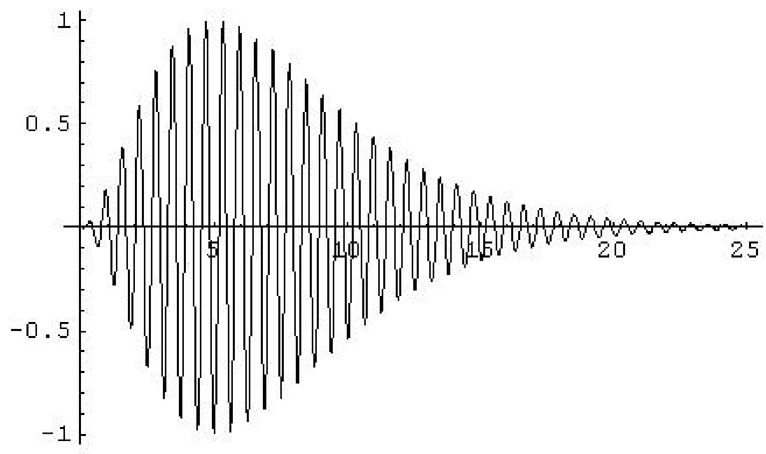

Figure 3: The seismic impulse.

4 THE APPROXIMATE SOLUTION OF THE EQUATION (1.1)

In Sections 1–3 the problem is formulated. We seek its solution satisfying to the initial condition $u(x,0) = 0$ as a partial sum of the Fourier series of eigen functions $U_k(x)$ of the linear boundary value problem

$$u(x,t) = \sum_{k=1}^{K} U_k(x) q_k(t). \tag{4.1}$$

The boundary value problem for natural modes $U_k(x)$ and for the corresponding natural frequencies ω_k is to be obtained from equation (1.1) and boundary conditions (1.6) after replacements

$$\frac{\partial^2}{\partial t^2} = -\omega^2, \quad f = a = \beta_0 = \beta_u = \beta_\varphi = 0, \quad \psi = \phi_u = \phi_\varphi = 1. \tag{4.2}$$

For $\omega_k \neq \omega_n$ the natural modes satisfy to the orthogonality condition

$$\int_0^H \rho U_k U_n dx + m_0 U_{k0} U_{n0} + J_0 U_k'(0) U_n'(0) + m_1 U_{k1} U_{n1} + J_1 U_k'(H) U_n'(H) = 0, \tag{4.3}$$

Where by the trait the derivative with respect to x is denoted.

At $n = k$ the left side of the equality (4.3) gives the equivalent mass M_k corresponding to the k^{th} natural mode

$$M_k = \int_0^H \rho U_k^2 dx + m_0 U_{k0}^2 + J_0 (U_k'(0))^2 + m_1 U_{k1}^2 + J_1 (U_k'(H))^2. \tag{4.4}$$

To construct the equations for unknowns $q_k(t)$ in (4.1) we use the Bubnov–Galerkin method according which the work of active forces and of inertia forces on the displacement $U_k(x)$ is equal to zero. As a result we get

$$M_k \left(\ddot{q}_k + \beta_k \omega_k \dot{q}_k + \omega_k^2 q_k \right) + M_k^a a = Q_v + Q_s + Q_F, \quad k = 1, \ldots, K. \tag{4.5}$$

whis

$$M_k^a = \int_0^H \rho U_k \, dx + m_0 U_{k0} + J_0 U_k'(0) + m_1 U_{k1} + J_1 U_k'(H). \tag{4.6}$$

By dots the derivatives with respect to time t are denoted. The summand $\beta_k\omega_k\dot{q}_k$ in (4.5) (β_k are the dimensionless resistant coefficients) takes into account the resistance of the k^{th} natural mode. Here we make the additional assumption that the decreases of various natural modes are not connected to each other. As Q_k^v, Q_k^s Q_k^F we denote the nonlinear generalized forces which are connected with the resistance of water, with the plastic properties of the rod, and with the interaction forces between the foundation and the ground respectively

$$
\begin{aligned}
&Q_k^v = -\int_0^H C_v\gamma R(x)v|v|U_k(x)dx, \quad v = \int_0^t a(t)\,dt + \sum_{i=1}^K U_i(x)\dot{q}_i(t),\\
&Q_k^s = -\int_0^H J(x)(1-\psi(\kappa))U_k''(x)dx, \quad \kappa = \sum_{i=1}^K U_i''(x)\dot{q}_i(t),\\
&Q_k^F = c_u u_*(1-\phi_u(u_*))U_{k*} + c_\varphi u'(0)(1-\phi_\varphi(u'(0)))U_k'(0), \qquad (4.7)\\
&\quad u_* = \sum_{i=1}^K (U_i(0) - h_*U_i'(0))q_i(t), \quad U_{k*} = U_k(0) - h_*U_k'(0),\\
&\quad u'(0) = \sum_{i=1}^K U_i(0)q_i(t).
\end{aligned}
$$

At $Q_v = Q_s = Q_F = 0$ the system (4.5) divides to K independent equations.

5 NUMERICAL RESULTS.

We fix the values of structure parameters and study the series of values of parameters of the seismic acceleration in (3.1).

We take the following parameters given in SI. The dimensions of structure and radii of the rod layers: $H_1 = 235$, $H = 250$, $r_{10} = 4.9$, $r_{20} = 4.95$, $r_{30} = 5.95$, $r_{40} = R_0 = 6$, $R_1 = 5$. The densities of water, steel, and concrete: $\gamma_w = 10^3$, $\gamma_s = 7.85 \cdot 10^3$, $\gamma_c = 2.2 \cdot 10^3$. The Young modules and the yield limits of steel and concrete: $E_s = 2.06 \cdot 10^{11}$, $E_c = 0.131E_s$, $\varepsilon_s = 0.00134$, $\varepsilon_c = 1.16\varepsilon_s$. The mass parameters and the distances to the mass centers of bodies *0* and *1*: $m_s = 2.32 \cdot 10^7$, $m_0 = 10^7$, $m_1 = 10^7$, $J_0 = 15m_0$, $J_1 = 10m_1$, $h_0 = 3$, $h_1 = 3$. The foundation parameters: $c_u = 10^7$, $c_\varphi = 10^{12}$, $u_{cr} = 10$, $\varphi_{cr} = 0.1$, $m_u = 2$, $m_\varphi = 2$. The visco-elastic resistant parameters: $\beta_k = 0.1,\ k = 1,2,3,4$. The resistant coefficient of motion in water $C_v = 2.2$.

We take $K = 6$. In Table 1 there are the first six natural frequencies ω_k, the corresponding periods, and the mass parameters $M_k,\ M_k^a$.

k	1	2	3	4	5	6
ω_k	0.256	0.688	3.117	8.158	15.89	26.39
T_k	24.56	9.15	2.02	0.77	0.40	0.24
$M_k \cdot 10^{-7}$	2.58	7.20	7.19	19.30	66.13	931.00
$M_k^a \cdot 10^{-7}$	1.32	-0.22	2.52	1.42	4.91	46.28
M_k^a/M_k	0.514	-0.031	0.351	0.074	0.071	0.050

Table 1: The natural frequencies and the mass parameters.

In the last line of Table 1 there is parameter M_k^a/M_k, which characterizes the level of the k^{th}

eigen mode excitation. We see that the modes with $k = 1$ and $k = 3$ are excited larger than the other ones.

We bound with the analysis of the seismic impulses of the form shown in Fig. 3 and change the maximal amplitude $A_{max} = \max_t |a(t)|$ and frequency ν. Then the parameters in (3.1) are $A = 0.295 A_{max},\ \alpha = 0.4$, and the parameter θ is to be found from (3.3).

We change the frequency ν of seismic impulse in the limits $0.5 \le \nu \le 30$ (1/c), and will increase A_{max} nd fix the value, for which the rod material turns in the nonlinear area according the condition (2.6). Calculations show (see also Table 2) that in all studied cases the condition when the concrete is extended is critical. Such deformations are inadmissibe from the point of view of the concrete strength.

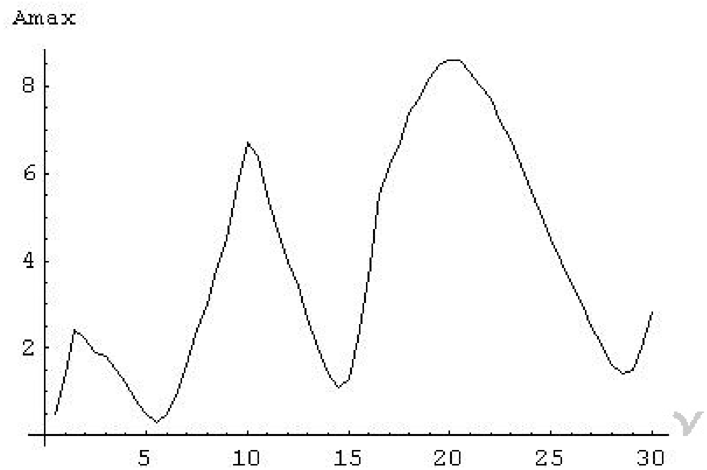

Figure 4: The boundary of the parameters (ν $(1/c)$, A_{max} (m/c^2)) area, below which the concrete is compressed only.

In Fig. 4 the boundary of parameters area in the plane (ν, A_{max}), upper which the concrete may be stretched. The more dangerous are impulses with the lower values of the frequency ν, which are not typical for the seismic ones. For seismic impulses the values $10 \le \nu \le 300$ [3]. Here the impulses with $\nu \le 30$ are studied because for $\nu > 30$ the used beam model is doubtful.

k	ν	A^{k0}_{max}	$u^{k0}(H_1)$	A^{ks}_{max}	$u^{ks}(H_1)$	A^{kc}_{max}	$u^{kc}(H_1)$
1	10	6.7	0.06	18.1	0.24	24.4	0.28
2	15	1.3	0.01	14.4	0.07	14.9	0.08
3	20	8.6	0.02	23.7	0.07	33.3	0.10

Table 2: The amplitudes of transition in the nonlinear area of the rod deformations.

In Table 2 three values of ν are studied, and the values ν_1 and ν_3 correspond to the points of the relative maximum of the curve $A_{max}(\nu)$ (Fig. 3), and ν_2 corresponds to the minimal point. As A^k_{max} and $u^k(H_1)$ the maximal amplitude of the seismic impulse (in m/c^2) and the maximal deflection of the upper body *1* (in m). Indexes $0,\ s,\ c$ correspond to appear extended deformations in concrete, to appear the yield of steel and to appear the yield of concrete at compression. It is clear that the values A^{ks}_{max} and A^{kc}_{max} are too large for seismic impulses. Therefore at design it is necessary to pay the main attention to the concrete strength.

6 CONCLUSIONS

In book [1] there is the investigation of the platform dynamics under action of surface gravity waves. In this problem the typical frequency of waves in some cases is close to the first natural frequency of structure. That is why it is possible in [1] to reduce the system to the single equation of the first order (with $K = 1$). At the seismic excitation the typical frequencies are much larger than the first natural frequency of structure. We take $K = 6$ but for $\nu > 30$ the exactness of the system (1.1) is not enough (see the values ω_k in Table 1) and it is desirable to use the more complex mathematical models of this structure.

Plans of the following investigation of this structure may consist in the complification of model by introducing the shear rod deformations. It is important in one side if we want to study the seismic excitations with the more high frequencies, and in the other side to take an attention that the concrete badly hold the shear stresses. Also it is necessary to study the vertical seismic excitation and the corresponding vertical vibrations of the rod. At last it is desirable to study the simultaneous action of the horizontal and the vertical seismic excitation and the interaction of the bending and the longitudinal rod vibrations.

Here we take the model seismic impulse. It seems that the real random seismic impulse with the same maximal amplitude leads to the smaller amplitude of the rod vibrations. Nevertheless it is desirable to study the structure reaction under action of the random seismic impulse with the given spectral density.

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