ESTIMATION OF THE SEISMIC RELIABILITY OF BUILDINGS USING INCREMENTAL DYNAMIC ANALYSIS METHOD OF THE SECANT STIFFNESS DEGRADATION INDEX (*IDAM-SSDI*). PART II

Marco Antonio Montiel Ortega[1], Guillermo Arturo Díaz de León Piña[1]

[1]Universidad Autónoma de Querétaro, México
Cerro de las Campanas s/n; Santiago de Querétaro, Qro., México,
mmo02@hotmail.com; gmo@uaq.mx

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Abstract. *Presented herein is a methodology that enables us to obtain in an efficient manner the seismic reliability of buildings for different levels of seismic intensity, taking into account the uncertainties associated with the properties of the structural system considered and with the seismic registers. This methodology is developed by means of Incremental Dynamic Analysis of the Secant Stiffness Degradation Index (IDA-SSDI), whose results enable evaluation of the intensity of system collapse in an easy and efficient manner in order to evaluate the reliability by means of Cornell's β index and a Z safety margin. The methodology is applied to a sample of eight 10-story buildings taken from Monte Carlo's simulation and 40 accelerograms registered in the Valley of Mexico City. The buildings simulated are modeled in the computer program DRAIN 2D for non-linear analysis.*

1 INTRODUCTION AND METHODOLOGY

The objective of this research is to collaborate in the mitigation of risks of a natural phenomenon that has been considered very important due to its effects and its ability to cause the end of the life cycle of a structure in a few seconds as in a seismic movement. The collapse of a structure brings with it the loss of human life and property which is associated with this type of natural disaster. The analysis of the probability of structural failure or of its complement, structural reliability, which has its beginnings in the 1960's, has served as a base in the development of structural engineering design methods that are presently being utilized as a guide in the creation of codes and manuals employed for the design and construction of secure structures by taking into account natural phenomena and uncertainties throughout the world.

Estimating of this structural reliability with reference to collapse occasioned by some seismic action is based on the capacity of lateral deformation in such a way that this collapse occurs when the maximum amplitude of the structural seismic response, or demand, surpasses the structural capacity. The deformation capacity concept as an indicator of the variable that determines the condition of collapse has severe limitations in its practical use when being used in quantitive terms.

These limitations range from sensitivity at the moment of impeding the collapse to certain variables, such as: the lateral deformation pattern of the system and the reductions of resistance and stiffness associated with the damage produced by previous response cycles corresponding to previous seismic occurrences, or by other causative agents of damage such as differential settlements. All these factors have inspired many new investigative endeavors to develop alternative criteria oriented to obtaining probabilistic estimations of seismic intensity values that lead to system collapse (Y_C).

One of the criteria utilized is the *Incremental Dynamic Analysis Method (IDAM)* proposed by Vamvatsikos and Cornell [1], which is based on the collapse intensity estimation *(Yc)* with the help of an adequate scale factor that should be applied to the accelerogram. The possibilities of visualizing the evolution of the seismic response amplitudes in the manner in which the intensity grows and of observing the values reached by that intensity before the appearance of the seismic response amplitude outside of its limits, constitutes an important advantage of this method. For these reasons, in this article, the collapse intensity *(Yc)* is obtained by using the *Incremental Dynamic Analysis Method* but applied for the first time to the *Secant Stiffness Degradation Index (IDAM-SSDI)*. This index is arrived at by the following equation [2]:

$$D(y) = 1 - \frac{K}{K_0} \tag{1}$$

Where *(y)* is the intensity of the seismic movement, K_0 is the value that K acquires when linear, and K is the value of the degraded secant stiffness adopted by the system at the moment when the lateral displacement in the roof reaches its maximum value. In this way, the collapse intensity *(Yc)* is obtained when *D(Yc)=1*.

On the other hand, the seismic reliability level is obtained by means of Cornell's [3] reliability index β, as follows:

$$\beta(y) = \frac{m_Z(y)}{\sigma_Z(y)} \tag{2}$$

Where $m_Z(y)$ is the median value of the building security margin Z before an earthquake with an intensity equal to *(y)*, and $\sigma_Z(y)$ is the standard deviation of the said security margin. For the estimation of the index $\beta(y)$, for the present work, an alternative approach proposed by Díaz and Esteva, [4] was used, where the safety margin $Z(y)$ is defined as the natural loga-

rithm of the rate of *(Yc)* and of the intensity of the ground movement *(y)* that act upon the system.

$$Z(y) = ln\frac{Y_C}{y} = lnY_C - lny \tag{3}$$

Consequently, collapse occurs when *Z(y)=0* for *D(y)=1*. The reliability index is defined as:

$$\beta(y) = \frac{E(lnY_C) - ln(y)}{\sigma(lnY_C)} \tag{4}$$

Where *E(lnYc)*, is the expected value of *lnYc*.

2   ANALYZED STRUCTURES

The aforementioned methodology is applied to a sample of eight 10-story buildings from the Monte Carlo simulation taking into account the uncertainties associated with the concrete strength *(f'c)*, the steel yield *(fy)*, the live loads *(Wvmax)* and 40 accelerograms registered in the valley of Mexico City. The geometric properties of the original building are shown in Figure 1a. The building was designed in accord with the Mexico City Seismic Design Code [5,6] for office use. The fundamental period of the building is $T_o = 1.17s$ and has a yield strength coefficient of *Cy=0.4*, [7].

The simulated buildings are modeled in the DRAIN 2D program modified by Campos and Esteva [8], for its linear analysis. Each building contains an exterior and interior structural frame connected by a rigid floor diaphragm. The coupling between the frames was taken into account by means of a two-dimensional structural model connected by hinged links, see Figure 1b. The frames were constituted by flexural beams and columns. The moment-rotation ratio for each element was calculated assuming the model for confined concrete originally proposed by Kent and Park [9] and modified by Park et al [10]. The axial stress-strain ratios corresponding to the steel bars were represented by means of Mander model [11]. The hysteretic structural behavior was assumed to be bilinear with the ratio of the post-yielding to the initial stiffness equaling 3.0%.
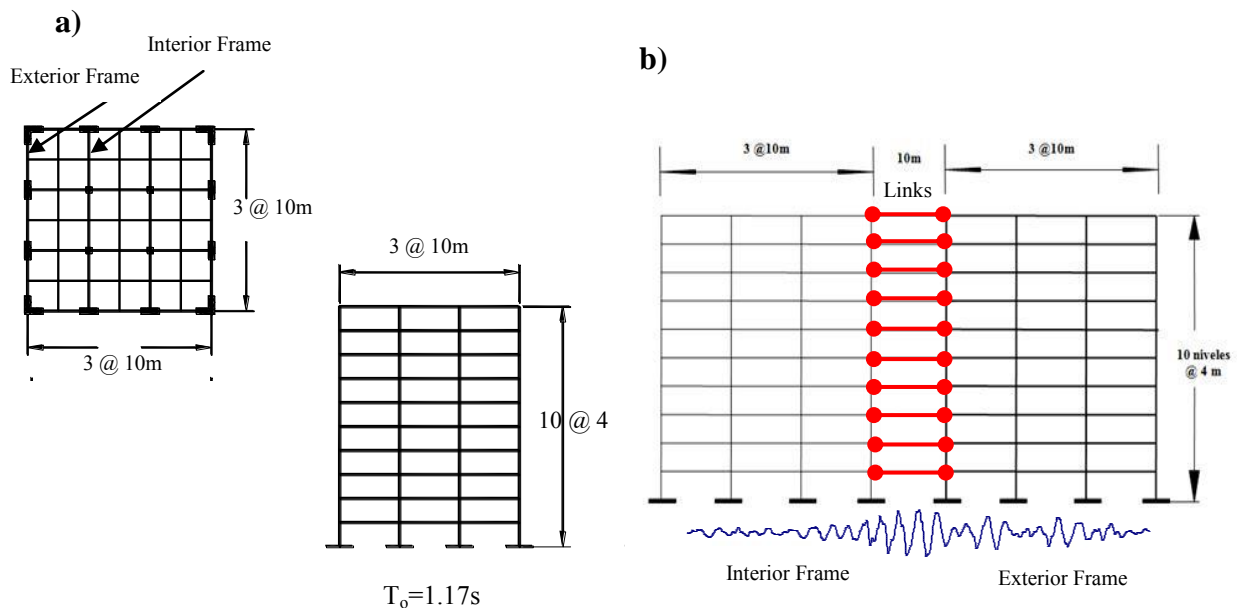

Figure 1. a) Original building plan and elevation and b) two-dimensional model used for non-linear analysis.

In Table 1 the eight models are shown that consider the uncertainties of the mechanical properties of the materials (f'_c and f_y) and in the maximum live loads applied (**$W_{vmáx}$**). The nominal values *(MM)* and means *(MN)* are also shown. In this table the nomenclature (*M1-M8*) is used to denote the models with uncertain values for mechanical properties of the materials and applied live loads. *(M9)* is used for the model with median values and *(M10)* is used for the model with nominal values. The uncertain values of each parameter (*f'_c, f_y* and *$W_{vmáx}$*) are obtained through simulations [12].

Frame	f'_c (kg/cm²)	f_y (kg/cm²)	$W_{vmáx}$ (kg/m²)
M1	308	5090	78
M2	246	4445	134
M3	287	3970	40
M4	234	4816	89
M5	250	4055	57
M6	199	5553	99
M7	330	5974	104
M8	304	4407	67
M9=MM	**268**	**4680**	**75**
M10=MN	**250**	**4200**	**180**

Table 1. f'_c ,f_y y $W_{vmáx}$ values used in frame analysis.

3   SEISMIC MOVEMENTS USED FOR THE ANALYSIS

The analysis of seismic reliability requires the use of registers that can adequately reflect the dynamic characteristics and the energy content of the seismic movements expected to occur at the construction site. For this analysis, forty narrow-band earthquakes registered in the lake region of Mexico City were used. All the seismic movements were recorded during subduction events with epicenters located on the Pacific coast of Mexico, see Figure 2a, which present magnitudes *(Mw)* from 6.0 to 8.1 grades. The corresponding elastic strength spectra of pseudo-acceleration are shown in Figure 2b, for a critical damping percentage (ξ) of 5%. It can be seen that the seismic movement spectra present dominant periods of between 1.5 and 2.2s.
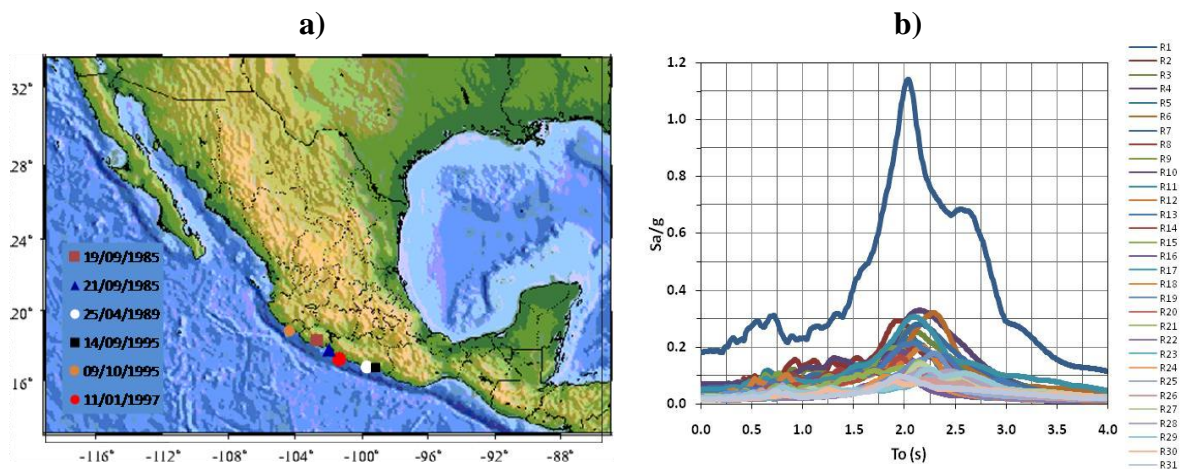

Figure 2. a) Subduction earthquake epicenters used for the analyses and, b) the pseudo-acceleration corresponding elastic spectra for 5% damping.

4 INCREMENTAL DYNAMIC ANALYSIS OF THE SECANT STIFFNESS DEGRADATION INDEX *(IDA-SSDI)*.

The *Incremental Dynamic Analysis Method* of the *Secant Stiffness Degradation Index (IDAM-SSDI)* was used to obtain the probabilistic estimations of the seismic intensity values that lead to system collapse in order to evaluate the structural reliability. These results are shown in Figure 3 associated with the 40 earthquakes and eight 10-story frames used for the analysis, taking into account the uncertainties associated with the earthquakes and the mechanical properties of the building materials. In this figure, we graph the seismic intensity level measured by means of the pseudo-acceleration spectral corresponding to the fundamental period of the structure *(T_o=1.17s)* for a 5% critical damping *(Sa/g)* divided by the acceleration of gravity *(g)*, which in turn has been affected by an adequate scaling factor in order to bring about the collapse of the structure.

In the horizontal axis the performance of the structure is graphed measured by the Secant Stiffness Degradation Index *D(y)*, which is an index proposed as an indicator of structural damage caused by an earthquake according to increase in intensity. The seismic effects decrease the capacity of the building leaving the structural system in a state of greater vulnerability and eventually leading to collapse.
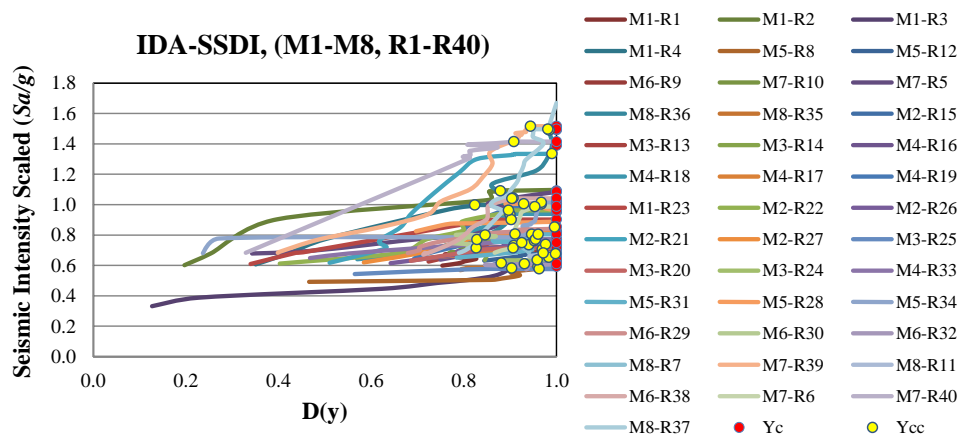

Figure 3. Incremental Dynamic Analysis of the Secant Stiffness Degradation Index (*IDA-SSDI*) for 40 earthquakes and the eight 10-story buildings being studied. Yellow circles indicate the intensity approaching collapse (Y_{CC}) and red circles indicate the collapse intensity (Y_C).

The curve forms in Figure 3 vary depending on the earthquake, the frame and the grade of the non-linear structure. These curves are obtained by scaling the seismic intensity until a state of collapse occurs. This is indicated in the figure by means of red circles *(Y_C,)*, which occur when the secant stiffness degradation value adopted by the system at the moment when the lateral displacement in the roof reaches its maximum value, that is zero, *(K=0)*. That is to say, when the Secant Stiffness Degradation Index is equal to the unit *(D(Y_c)=1)*, which corresponds to a base shear in terms of very low or zero capacity. One moment before the state of collapse occurs, the seismic intensities nearing collapse are obtained, which are indicated in the figure with yellow circles *(Y_{Cc})*, varying from *0.823≤ D(y)<1.0*. Using *D(y)* as an index of collapse is very useful because it eliminates the difficulty of defining the collapse state from the maximum displacement of the roof. By means of *IDAM-SSDI* analysis, valuable information can be obtained regarding the behavior of the structural system ranging between *0.8≤d<1.0* associated with the limit state near collapse.

The statistical values of the parameters corresponding to the average and to the standard deviation of collapse and near collapse for the intensity and the damage index are shown in Table 2.

	Y_{Cc} Sa/g	Y_C Sa/g	$D(Y_{Cc})$	$D(Y_C)$
Average	0.8832	0.8898	0.9308	1.00
Standard deviation	0.2610	0.2620	0.0470	0.00

Table 2. Statistical parameters of the limit state near collapse and at collapse.

There are no significant differences between the intensities Y_{Cc} and Y_C needed in order to evaluate reliability, which is not the case for $D(Y_{Cc})$ and $D(Y_C)$, whose values vary depending on the structural sensitivity in the zone near collapse.

The Incremental Dynamic Analysis Method results attained for each frame and earthquake by means of the scale factors used from a state near collapse to a state of collapse intensity (Y_C) are presented in Figure 4. The curves with greater slope require a Scaling Factor (SF) greater than that required for curves with lesser slope. It can be observed that for some earthquakes, very low scale factors of 3 or 4 are needed; for other earthquakes, factors as high as 70 or 80 are needed, which means that these factors vary due to the uncertain nature of earthquakes and the variations in mechanical properties of the materials of the buildings under investigation.
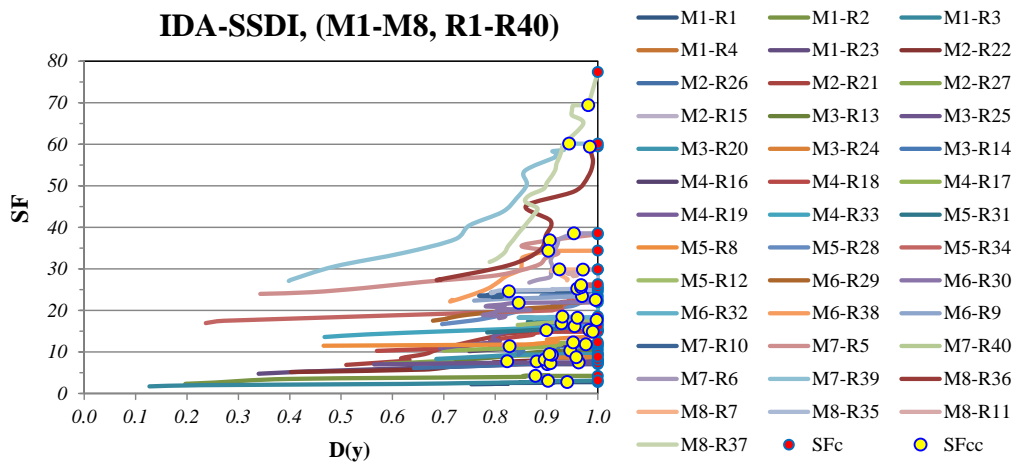

Figure 4. Incremental Dynamic Analysis Method of the Secant Stiffness Degradation Index *(IDAM-SSDI)*, depending on the scale factor *(SF)*.

It can also be observed that the seismic collapse intensity values range from $0.594g$ as a minimum value in Figure 3 for the Frame *M8* and the earthquake *R35* with a 25.21 scaling factor *SF* in Figure 4, up to the maximum value of $1.52g$ found for Frame *M7* for the seismic record *R39* with a 60.17 *SF*.

Figures 5a, b, and c, show the effect of the scale factor *(SF)* has on the overall structural response and is represented by means of the base shear and maximum roof displacement hysteresis cycles. From this the secant stiffness *(K)* required to obtain the damage index *D(y)* is obtained. This stiffness is obtained from the straight line slope that connects the origin with

the maximum roof displacement. The slope is obtained by dividing the base shear from the corresponding maximum roof displacement.

In Figure 5a, it can be observed that for a *SF=17.51* for this earthquake and frame *(R34, M5)*, there exists an almost linear behavior, such that the secant stiffness *K=7196.92 tons/m* is similar to the elastic stiffness: $K_0=9882.86$ *tons/m*, so that the result for the damage index is low: *D(y)=0.272*. This is to say, a structure that is practically elastic.

In Figure 5b, for *SF=23*, the nonlinear behavior is significant, therefore the secant stiffness is low: *K=384.40 tons/m*, resulting in a damage index close to one: *D(y)=0.9611*.

In Figure 5c, for an *SF=26.30*, as can be observed, the structure has collapsed, and displacements is considerable, giving a value of *K=0*; and therefore, a damage index of *D(y)=1*.

Of course, in the previous cases, there was intermediate *SFs,* which made possible the observation of the evolution of the damage index *D(y)* as can be seen in Figures 3 and 4.
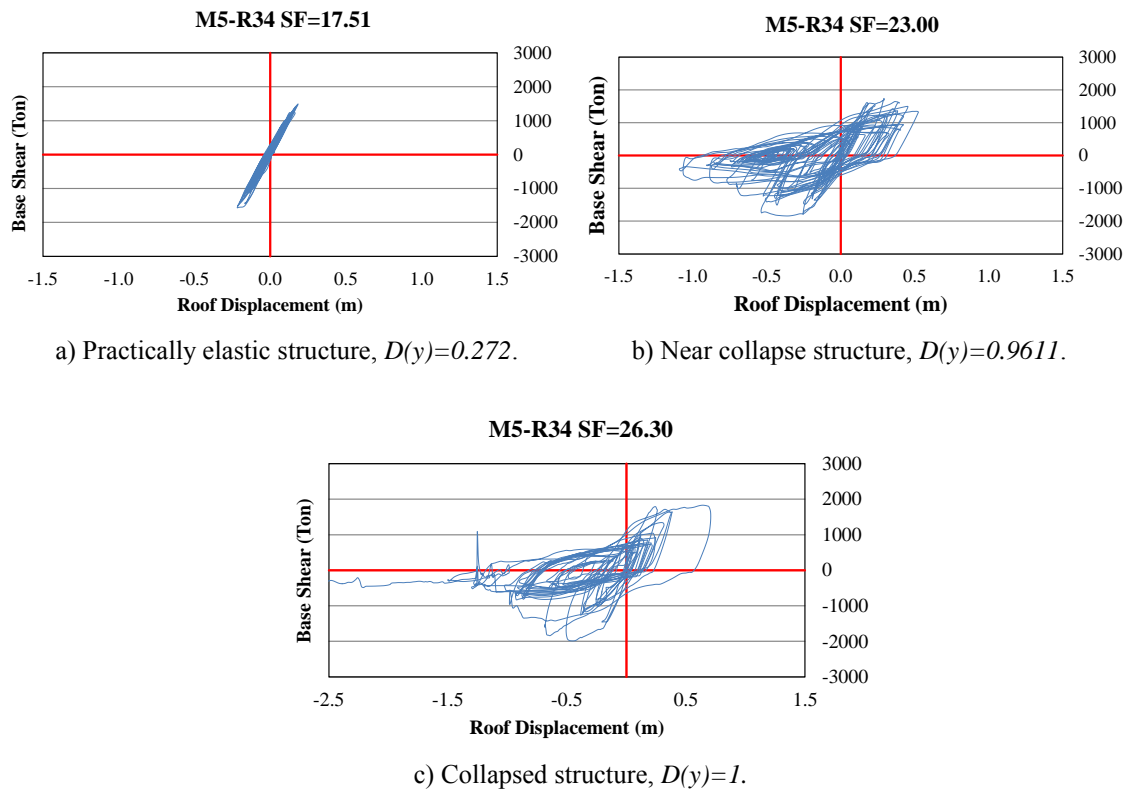

a) Practically elastic structure, *D(y)=0.272*.

b) Near collapse structure, *D(y)=0.9611*.

c) Collapsed structure, *D(y)=1*.

Figure 5. Global response hysteresis cycles for different Scale Factors *(SF)*.

The elastic stiffnesses *(K₀)* of the simulated frames were obtained from the Pushover analyses, which are shown in Figure 6.
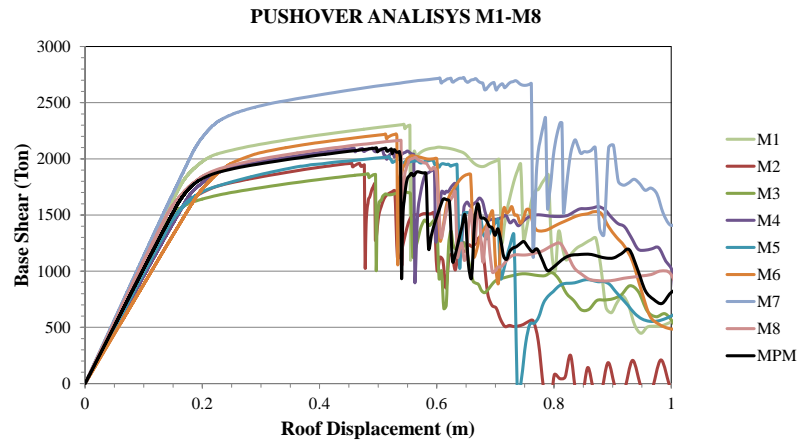

PUSHOVER ANALISYS M1-M8

Figure 6. Pushover analyses for eight 10-story frames (M1-M8). The black line corresponds to the frame with mean mechanical proprieties (MPM).

5   RELATIONSHIP BETWEEN THE DAMAGE INDEX *D(y)* AND THE SAFETY MARGIN *Z(y).*

Another way of seeing how structures vary as they near to collapse, when *D(y)* reaches its maximum value *D(y)=1*, is by means of the relationship between the damage index *D(y)* and the safety margin *Z(y)*, which is a function of the collapse intensity *(Y_C)*, and the earthquake intensity *(y)*, in other words: *Z(y)=ln(Y_C)-ln(y)*, see Equation 3. The value pairs for these variables can be seen in Figure 7 for the different frames and earthquakes analyzed. It can be observed that as the safety margin decreases the structure approaches collapse when *D(y)=1* and *Z(y)=0*. This tendency can be better seen by means of linear regression represented by the blue curve representing the expected value of the safety margin *E(Z)*. It can be noted that this curve begins in an area where the values represent a low structural vulnerability, values of *Z>0* far from collapse and values for *d* close to zero with linear elastic structural behavior, to the collapse values when *(D=d=1, Z=z=0)*, or better, when *lnD=0* and *Z=0*. In the event that *Z(y)* is negative, *Z(y)<0* indicates that the structural collapse intensity *(Y_C)* was surpassed by the earthquake intensity *(y)*, which is to say that it has already collapsed *D(y)=1*, as can be seen in Figure 7.

Figure 8 shows the deviation trend for the safety margin *Z* related to the damage index values *D(y)* and its trend curve which is produced by linear regression. It may be noted that in general the values are low as the structure approaches collapse *D(y)⇒1*, the variance *σ²(Z)* tends to zero.

M1-M8, R1-R40
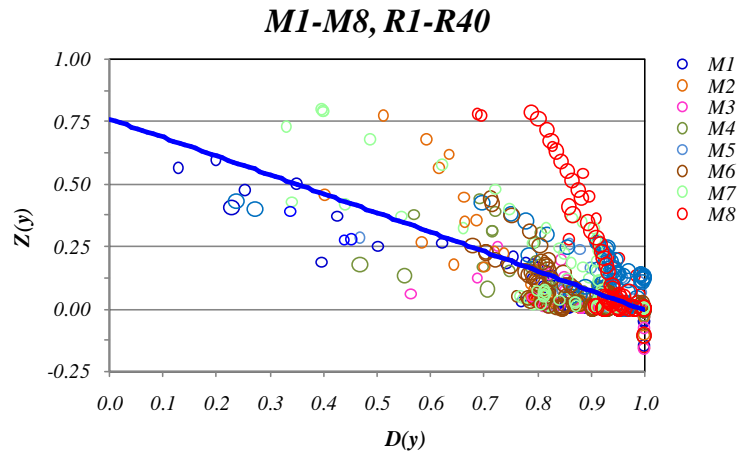

Figure 7. Relationship between the damage index $D(y)$ and the safety margin $Z(y)$. Blue line: expected value of $E(Z)$.
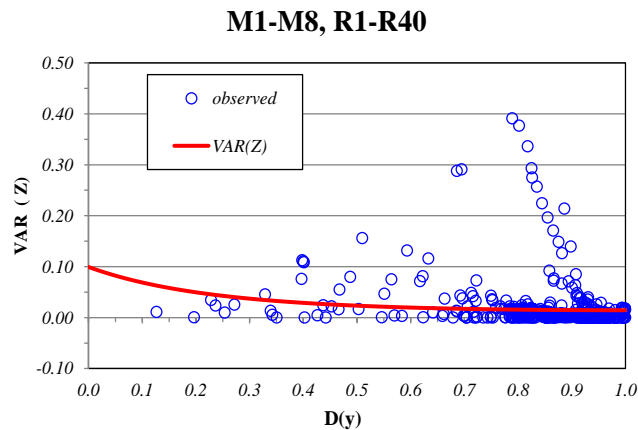
M1-M8, R1-R40

Figure 8. Mean square error values used to estimate the variance of Z and its trend line in red.

6   OBTAINING SEISMIC STRUCTURAL RELIABILITY: $\beta(y)$ *vs* $Z(y)$.

With the collapse intensities (Y_C) obtained from the *IDA's,* the seismic reliability of buildings in terms of the safety margin Z in Figure 9 is obtained. The straight lines of the figure are obtained using Cornell's index $\beta(y)$ (vertical axis) corresponding to the eight 10-story frames and 40 earthquakes used for the analyses and how they relate to the safety margin for $Z(y)$ (horizontal axis), from the state of system collapse when $Z(y=Y_C)=0$, to values of $Z(y<<Y_C)=1$, when the intensity of the earthquake (y) is much lower than that of collapse (Y_C), when the structure is elastic. As can be seen, their relationship is linear, so a greater safety margin $Z(y)$ in the system corresponds to a higher reliability index $\beta(y)$ in the structure.
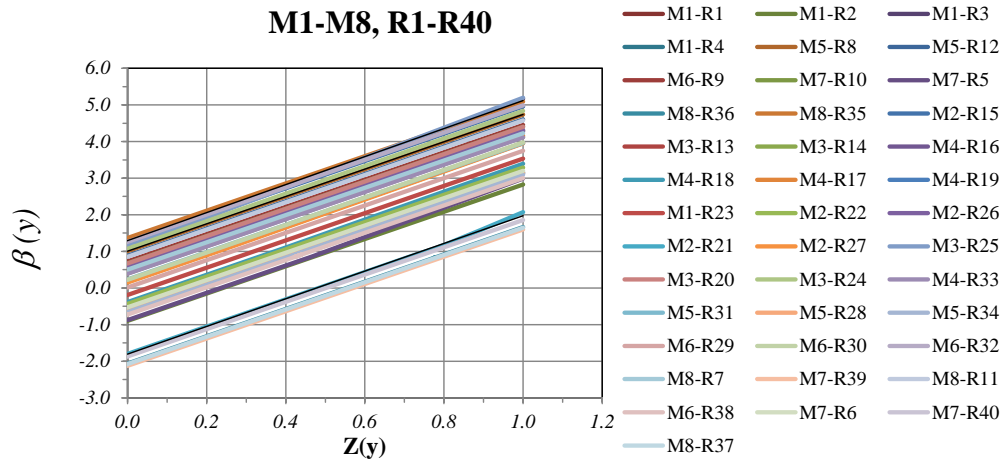
Figure 9. Cornell's Reliability Index $\beta(y)$ for different values of the Safety Margin $Z(y)$ for the eight 10-story frames of Monte Carlo simulated and 40 earthquakes used in the analyses.

With lower resistance in the frame and higher intensity of the earthquake, a straight line with a lower reliability index in Figure 9 is obtained. For the same $Z(y)$, different reliability levels are obtained depending on the level of uncertainties in the frame and the earthquake in question.

For purposes of design or evaluation, a suitable reliability index $\beta(y)$ must be positive, reaching values of up to 5 as shown in Figure 9 for a safety margin $Z(y)=1$, indicating a structure with a completely elastic behavior. A negative reliability index is an undesirable index, which indicates a high probability of collapse and this happens when the expected value of the natural logarithm of collapse intensity is reached and/or surpassed by the intensity of earthquake; in other words: $y_i \geqslant E(lnY_C)$. In this way, a safety margin $Z(y)$ would be recommended as long as it is high enough for a determined limit state for a positive reliability index. For example, for the straight lines in the lowest part of the figure, the minimum $Z(y)$ required should be greater than 0.6 to obtain a positive index. This happens to the frame of least resistance $M8$ and for an intense earthquake $R37$, and for $M7\text{-}R39$ and $M8\text{-}R36$.

The forty straight lines corresponding to each building and earthquake ($M\text{-}R$) in Figure 9 are all parallel to each other, consecuently, they share the same slope m, showing the differences between each one of these straight lines in the corresponding values for $\beta(Y_C)$ when $Z(Y_C)=0$. Due to the linear relationship of the variables, in general terms, the lines in the figure can be expressed as follows:

$$\beta(y)=\beta_{Z=0}(Y_C)+m\bullet Z(y) \tag{5}$$

Where: $\beta_{Z=0}(Y_C)$ is the ordinate at $x=0$, which is to say that the value of $\beta(y)$ when collapse occurs: $Z(y)=0$.

Moreover, a particular expression can be found for obtaining the reliability index, or the safety margin that we wish to assign it to the structure in question considering that $\beta_{z=0}(Y_C)$ is known or easily obtained from the results already obtained, in this case for the 10-story buildings:

$$\beta(y)=\beta_{Z=0}(Y_C)+3.698\ Z(y) \tag{6}$$

Where: $\beta_{Z=0}(Y_C)$ indicates the level of uncertainties associated with the earthquake, the mechanical properties and the live loads of the building considered in this study. For the par-

ticular case in which the ordinate is zero: $\beta_{Z=0}(Y_C)=0$, the index $\beta(y)$ will have a relationship of 3.7 times the safety margin $Z(y)$.

7 CORNELL'S RELIABILITY INDEX $\beta(y)$ RELATED TO THE SECANT STIFFNESS DEGRADATION INDEX $D(y)$.

Another way of relating Cornell's reliability index $\beta(y)$ is based on the damage index $D(y)$ for a given intensity (y), such as is shown in Figure 10. To the degree that the reliability level decreases according to the damage index comes closer to collapse, that is to say for $D(y)=1$. Just as in Figure 9, when $\beta(y)$ is negative indicating that the average collapse intensity of has been surpassed by the intensity of the earthquake. If collapse intensity coincides with that of the average of all frames and earthquakes, then a reliability index of zero would occur: $\beta(y)=0$, as takes place for the safety margin: $Z(y)=0$, when $D(y)=1$, but by definition $\beta(y)$ takes into account the average of all collapse intensities, see Eq. 4 and not the collapse intensity of each earthquake, as $Z(y)$, see Eq. 3. In relation to this, this leads us to observe that in Figure 10 for a $\beta(y)=0$ or negative, a structural collapse has not necessarily been brought about; this happens only when the earthquake collapse intensity is close to average. In such a way a collapse could be obtained a $D(y)=1$ for a positive reliability $\beta(y)$ or a non-collapse $D(y)<1$ when reliability $\beta(y)$ is negative. Therefore, this figure is not the most suitable for establishing the relationship between the reliability and the damage index, but rather, it does provide a general idea of their values and facilitates the understanding of the behavior of these variables.
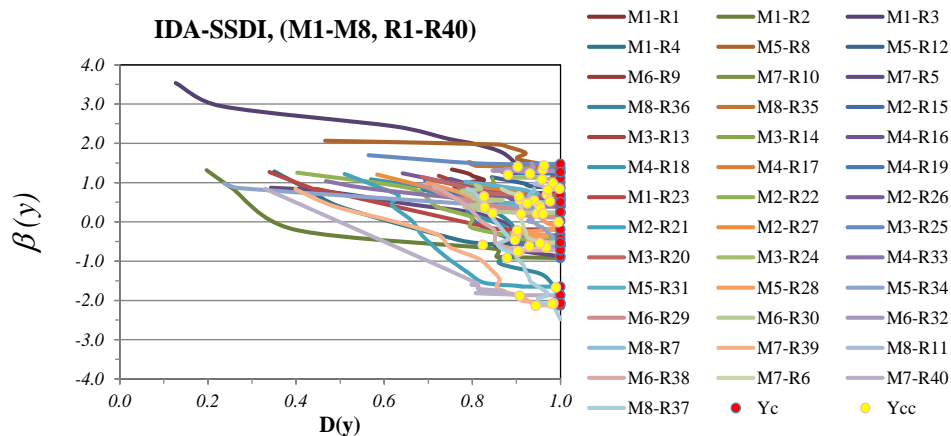

Figure 10. Cornell's Reliability Index $\beta(y)$ in relation to the damage index $D(y)$ corresponding to the eight 10-story frames and 40 simulated earthquakes used in the analyses.

8 CONCLUSIONS

- A more efficient methodology based on the Incremental Dynamic Analysis of Secant Stiffness Degradation Index (*IDA-SSDI*) for evaluating the reliability of buildings by means of Cornell's index $\beta(y)$ and the safety margin $Z(y)$ was developed.

- An expression, see Equations 5 and 6 that allows us to directly and simply evaluate the reliability of a 10-story building from a determined safety margin, (e.g. for the evaluation and/or structural design associated with a determined limit state) was obtained.

- For the first time Incremental Dynamic Analyses applied to the Secant Stiffness Degradation Index (*IDA-SSDI*) were obtained. These results allow easy and efficient evaluation of the collapse intensity (Y_C) of the system needed in order to assess the reliability.

- From Figure 9 suitable limits can be established for structural design and/or evaluation purposes of $\beta(y)$ and $Z(y)$, from 0.0 to 5.0 and from 0.6 to 1.0, respectively, for a 10-story building and depending on the limit state that one wishes to evaluate. We recommend studying other multi-story buildings, as a second phase.

9 ACKNOWLEDGMENTS

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