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EFFECT OF HIGHER ORDER SHEAR DEFORMATION ON THE NONLINEAR DYNAMIC ANALYSIS OF LAMINATED COMPOSITE PLATE UNDER IN-PLANE LOADS

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Abstract: *A nonlinear finite element method is adopted for the large displacement dynamic analysis of anisotropic plates under in-plane compressive loads. The analysis is based on the two-dimensional layered approach with higher order shear deformation theory with five, seven, nine, and eleven degrees of freedom per node, nine-node Lagrangian isoparametric quadrilateral elements are used for the discretization of the laminated plates. A complete bond between the layers is assumed (no delamination occurs). A consistent mass matrix is used in the present study. Damping property is considered by using Rayleigh type damping which is linearly related to the mass and the stiffness matrices. Newmark integration method and Harmonic acceleration method are used for solving the dynamic equilibrium equations. The effects of number of layers, damping factor, and number of degree of freedom per node on the large displacement dynamic analysis are considered. From the present study, noticed that the central deflection increase with increasing the degree of freedom per node.*

1.GENERAL

Certain civil engineering structures are designed to carry their own dead load plus superimposed loads which are immovable and unvarying with time, that is, superimposed static loads. In such cases, the stress analysis involves only principles of statics. More often the design of a civil engineering structure involves not only static loads but also superimposed loads which are either moving or movable and may vary with time as in superimposed dynamic loads. In such cases, the stress analysis properly should involve principles of dynamics to determine the effect of dynamic loading. However, in many of these cases, experience has shown that the dynamic effect makes a minor contribution to the total load which must be provided for the design and therefore the dynamic effect need not be evaluated precisely. In such cases, the dynamic effect may be handled by the use of an equivalent static load, or by an impact factor or by a modification of the factor of safety(3).

There have been a number of developments which have led to growing interest in a more precise evaluation of the effects produced by the dynamic portion of the loading. Among these are the imposition of more severe live load conditions (that is, machinery and vehicles moving at high speeds), the construction of high towers and long bridges involving more severe and important wind-loading conditions, the necessity of developing blast resistant constructions, and the desire to improve earthquake resistance of constructions. These are some aspects where it may be necessary to consider more precisely the response produced by dynamic loading.

The ability of thin-walled structures to absorb the energy of dynamic transient loading has led to its utilization for several classes of important structures, such as aerodynamic structures, power plant structure, bridge structures, etc. These types of structures are designed under these loads to maintain the overall structural integrity with irreversible deformation analysis. In the present study a computational modeling is developed for the nonlinear dynamic analysis of laminated composite plates using finite element method. The dynamic equilibrium Equation and the derivation of mass and damping matrices will be presented. A Newmark direct time integration method is adopted. In [**1993**], **Kommineni** and **Kant** presented a C^o-continuous finite element formulation of a higher order displacement theory for predicting linear and geometrically nonlinear behavior in the sense of von-Karman transient response of composite and sandwich plates. **Azevedo** and **Awruch** [**1999**] presented a geometric nonlinear dynamic analysis of plates and shells using eight-node hexahedral isoparametric elements. The main features of their study are: (1) the element matrices were obtained by using reduced integrations with hourglass control; (2) an explicit Taylor-Galerkin scheme was used to carry out the dynamic analysis by solving the corresponding equations of motion in terms of velocity components. **Tao, et al.** [**2004**] presented a simple solution of the dynamic buckling of stiffened plates under impact loading. Based on large deflection theory, a discretely stiffened plate model had been used. The tangential stresses of stiffeners and their in-plane displacements were neglected.

2. LAMINATED PLATE THEORIES

A laminated plate is a series of laminas bonded together to act as an integral structural element. Thus, a laminate is not a material but instead a structural element with essential features of both material properties and geometry. The stiffness and strength of such a composite material with structural configuration are obtained from the properties of the constituent laminas, and thus the macromechanical behavior of a laminate is the main topic of this section. The lamination so described can be considered as a single layer with "rule of mixtures" representation of the interaction between the multiple laminas in a plate or shell(6).

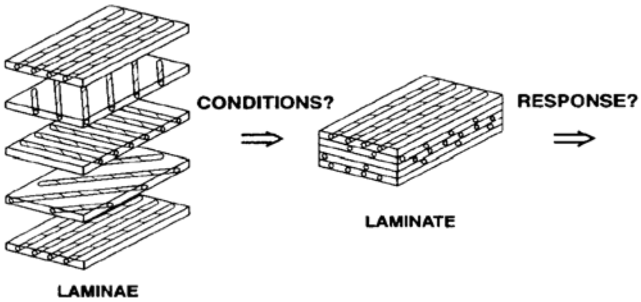

Figure (1): Laminated plate with several lamina orientations(6)

In the analysis of the laminated plates, there are two categories of theories, equivalent single layer and three dimensional elasticity theories. In the first category, the material properties of the constituent layer are smeared to form a hypothetical single layer whose properties are equivalent to through thickness integrated sum of its constituents, and this category contains classical lamination theory, first order shear deformation theory, and higher order shear deformation theory as will be given in the following section:

2.1 Classical lamination theory

Classical laminated plate theory is also often called "classical laminated theory (CLT)" which is based on the Kirchhoff-Love hypothesis for plates and shells (6). The assumptions of classical laminated plate theory are as follows:

1- The plate is thin. That is the thickness (h) is small compared to the other physical dimensions.
2- The displacements $u(x,y,z), v(x,y,z)$ and $w(x,y,z)$ are small compared to the plate thickness.
3- The in-plane strains $\varepsilon_x^o, \varepsilon_y^o$ and γ_{xy}^o are small compared to unity.
4- The transverse normal stress σ_z is negligible.
5- The transverse shear stresses τ_{xz}, τ_{yz} are negligible.

2.2 First order shear deformation theory (FSDT)

Timoshenko deep beam theory, which includes transverse shear deformation and rotary inertia effect, has been extended to isotropic plates by **Reissner** and **Mindlin**, and to laminate anisotropic plates by **Yang**, ***et. al***. and their theory, also called "First order shear deformation theory (FSDT)", takes into account the effect of transverse shear deformation and assume it constant through the plate thickness. Thus, a shear correction factor is used (3). The assumptions of First order shear deformation theory (FSDT) are as follows:

1- The in-plane displacements are linear functions of z (plane cross sections remaining plane after deflection).
2- The displacements $u(x,y,z), v(x,y,z)$ and $w(x,y,z)$ are small compared to the plate thickness.
3- The in-plane strains $\varepsilon_x, \varepsilon_y$ and γ_{xy} are small compared to unity.
4- The transverse normal stress σ_z is negligible.
5- The transverse shear stresses τ_{xz} , and τ_{yz} are considered to be constant through the plate thickness.

2.3 Higher order shear deformation theory (HSDT)

In general, a layered composite plate exhibits coupling between the in-plane displacements, transverse displacements and shear rotations. However, due to the low transverse shear modulus relative to the in-plane Young's modulus of each lamina, the transverse shear deformation effect is more pronounced in composite than in isotropic plates. Hence, several types of shear deformation theories have been introduced.

The higher order shear deformation theories are more efficient to represent the transverse shear deformation, through-thickness displacement and strains. The assumption of a higher order plate theory can also be used within the equivalent layer formulation [6].

The assumptions of higher order shear deformation are as follows:

1- The plate may be moderately thick.
2- The in-plane displacements $u(x,y,z), v(x,y,z)$ are cubic functions of z.
3- The transverse shear stresses τ_{xz} , and τ_{yz} are parabolic in z, no shear correction factor is necessary.
4- The in-plane stresses σ_x, σ_y, and τ_{xy} are cubic functions of z.
5- The normals to the mid-surface before deformation are straight, but not necessarily remain normal to it the mid-surface after deformation.
6- The transverse normal stress σ_z is negligible.

Figure (2) briefly shows, the basic difference between the classical and the first order theories with the higher order theories.

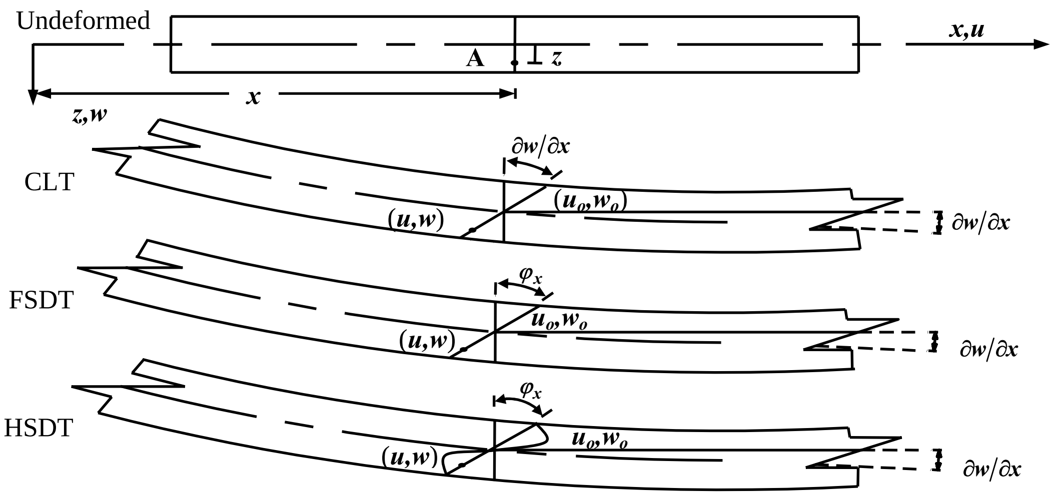

Figure (2): Kinematics of deformation of a plate in various plate theories[1]

All the prescribed theories are considered in the present study in order to study the effect of these theories on the accuracy and the time consumption in the analysis. In the present study, three types of displacement equations were considered. Firstly the displacement representation for this theory with five degrees of freedom per node is as follows:

$$
\begin{aligned}
u(x,y,z,t) &= u_o(x,y,t) + z\theta_x(x,y,t) \\
v(x,y,z,t) &= v_o(x,y,t) + z\theta_y(x,y,t) \\
w(x,y,z,t) &= w_o(x,y,t)
\end{aligned}
\qquad (1)
$$

in which t denotes the time; and u_o, v_o, and w_o are the components of the mid-plane displacements for a generic point (x,y,z) having displacements u, v, and w in x, y, and z directions, respectively. Here, θ_x and θ_y are rotations of transverse normals in the (xz) and (yz) planes, respectively.

The strain-displacement relations after differentiating Equation (1) are:

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = \varepsilon_x^o + z\kappa_x \\
\varepsilon_y &= \frac{\partial v}{\partial y} = \varepsilon_y^o + z\kappa_y \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy}^o + z\kappa_{xy} \\
\gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \varphi_x \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \varphi_y
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
&\varepsilon_x^o = \frac{\partial u_o}{\partial x},\ \varepsilon_y^o = \frac{\partial v_o}{\partial y},\ \gamma_{xy}^o = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \\
&\kappa_x = \frac{\partial \theta_x}{\partial x},\ \kappa_y = \frac{\partial \theta_y}{\partial y},\ \kappa_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \\
&\varphi_x = \theta_x + \frac{\partial w_o}{\partial x} \\
&\varphi_y = \theta_y + \frac{\partial w_o}{\partial y}
\end{aligned} \tag{3}$$

All the strains above are defined in the middle plane of the laminate and substitution these Equations into the stress-strain relations. Secondly the higher order shear deformation theory (HSDT) with seven degrees of freedom per node was considered. The strain expressions derived from the displacement field was considered by **Mallikarjuna,** and **Kant** [**1988**]**,** and by **Ali** [**2004**] with seven degrees of freedom per node as follows:

$$\begin{aligned}
u(x,y,z,t) &= u_o(x,y,t) + z\theta_x(x,y,t) + z^3\theta_x^*(x,y,t) \\
v(x,y,z,t) &= v_o(x,y,t) + z\theta_y(x,y,t) + z^3\theta_y^*(x,y,t) \\
w(x,y,z,t) &= w_o(x,y,t)
\end{aligned} \tag{4}$$

in which (u, v, w, θ_x, and θ_y) are defined previously, θ_x^* and θ_y^* are the corresponding higher order terms in Taylor's series expression and also defined at the middle plane. The strain-displacement relations after differentiating Equation (4) are:

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = \varepsilon_x^o + z\kappa_x + z^3\kappa_x^* \\
\varepsilon_y &= \frac{\partial v}{\partial y} = \varepsilon_y^o + z\kappa_y + z^3\kappa_y^* \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy}^o + z\kappa_{xy} + z^3\kappa_{xy}^* \\
\gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \varphi_x + z^2\varphi_x^* \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{w}{\partial y} = \varphi_y + z^2\varphi_y^*
\end{aligned} \tag{5}
$$

where the parameters ($\varepsilon_x^o, \varepsilon_y^o, \gamma_{xy}^o, \kappa_x, \kappa_y, \kappa_{xy}, \varphi_x, \varphi_y$) are defined previously.

$$
\begin{aligned}
&\kappa_x^* = \frac{\partial \theta_x^*}{\partial x},\ \kappa_y^* = \frac{\partial \theta_y^*}{\partial y},\ \kappa_{xy}^* = \frac{\partial \theta_x^*}{\partial y} + \frac{\partial \theta_y^*}{\partial x} \\
&\varphi_x^* = 3\theta_x^* \\
&\varphi_y^* = 3\theta_y^*
\end{aligned} \tag{6}
$$

Also, all the strains above are defined in the middle-plane of the laminate and substitution these Equations into the stress-strain relations.

Thirdly, Higher order shear deformation theory (HSDT) with nine degrees of freedom per node was considered. The strain expressions derived from the displacement field were considered by [Ali, 2004] with nine degrees of freedom per node as follows:

$$
\begin{aligned}
u(x,y,z,t) &= u_o(x,y,t) + z\theta_x(x,y,t) + z^2u_o^*(x,y,t) + z^3\theta_x^*(x,y,t) \\
v(x,y,z,t) &= v_o(x,y,t) + z\theta_y(x,y,t) + z^2v_o^*(x,y,t) + z^3\theta_y^*(x,y,t) \\
w(x,y,z,t) &= w_o(x,y,t)
\end{aligned} \tag{7}
$$

in which the parameters (u, v, w, θ_x, θ_y, θ_x^*, and θ_y^*) are defined previously, u_o^*, and v_o^* are the corresponding higher order terms in Taylor's series expression and they are also defined at the middle plane. The strain-displacement relations after differentiating Equation (7) are:

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = \varepsilon_x^o + z\kappa_x + z^2\varepsilon_x^{o*} + z^3\kappa_x^* \\
\varepsilon_y &= \frac{\partial v}{\partial y} = \varepsilon_y^o + z\kappa_y + z^2\varepsilon_y^{o*} + z^3\kappa_y^* \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy}^o + z\kappa_{xy} + z^2\gamma_{xy}^{o*} + z^3\kappa_{xy}^* \\
\gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \varphi_x + z\gamma_{xz}^o + z^2\varphi_x^* \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \varphi_y + z\gamma_{yz}^o + z^2\varphi_y^*
\end{aligned} \tag{8}
$$

where ($\varepsilon_x^o, \varepsilon_y^o, \gamma_{xy}^o, \kappa_x, \kappa_y, \kappa_{xy}, \kappa_x^*, \kappa_y^*, \kappa_{xy}^*, \varphi_x, \varphi_y, \varphi_x^*, \varphi_y^*$) are defined previously.

$$
\begin{aligned}
&\varepsilon_x^{o*} = \frac{\partial u_o^*}{\partial x},\ \varepsilon_y^{o*} = \frac{\partial v_o^*}{\partial y},\ \gamma_{xy}^{o*} = \frac{\partial u_o^*}{\partial y} + \frac{\partial v_o^*}{\partial x} \\
&\gamma_{xz}^{o*} = 2u_o^* \\
&\gamma_{yz}^{o*} = 2v_o^*
\end{aligned}
\tag{9}
$$

Also, all the strains above are defined in the middle-plane of the laminate. By substitution from Equation (8) into the stress-strain relations, after complete integration, the stress-resultant/strain relations of the laminate are as follows:

$$
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ N_x^* \\ N_y^* \\ N_{xy}^* \\ M_x \\ M_y \\ M_{xy} \\ M_x^* \\ M_y^* \\ M_{xy}^* \end{bmatrix}
=
\begin{bmatrix}
A_{11} & A_{12} & A_{16} & D_{11} & D_{12} & D_{16} & B_{11} & B_{12} & B_{16} & E_{11} & E_{12} & E_{16} \\
A_{12} & A_{22} & A_{26} & D_{12} & D_{22} & D_{26} & B_{12} & B_{22} & B_{26} & E_{12} & E_{22} & E_{26} \\
A_{16} & A_{26} & A_{66} & D_{16} & D_{26} & D_{66} & B_{16} & B_{26} & B_{66} & E_{16} & E_{26} & E_{66} \\
D_{11} & D_{12} & D_{16} & F_{11} & F_{12} & F_{16} & E_{11} & E_{12} & E_{16} & G_{11} & G_{12} & G_{16} \\
D_{12} & D_{22} & D_{26} & F_{12} & F_{22} & F_{26} & E_{12} & E_{22} & E_{26} & G_{12} & G_{22} & G_{26} \\
D_{16} & D_{26} & D_{66} & F_{16} & F_{26} & F_{66} & E_{16} & E_{26} & E_{66} & G_{16} & G_{26} & G_{66} \\
B_{12} & B_{12} & B_{16} & E_{11} & E_{12} & E_{16} & D_{11} & D_{12} & D_{16} & F_{11} & F_{12} & F_{16} \\
B_{12} & B_{22} & B_{26} & E_{12} & E_{22} & E_{26} & D_{12} & D_{22} & D_{26} & F_{12} & F_{22} & F_{26} \\
B_{16} & B_{26} & B_{66} & E_{16} & E_{26} & E_{66} & D_{16} & D_{26} & D_{66} & F_{16} & F_{26} & F_{66} \\
E_{11} & E_{12} & E_{16} & G_{11} & G_{12} & G_{16} & F_{11} & F_{12} & F_{16} & H_{11} & H_{12} & H_{16} \\
E_{12} & E_{22} & E_{26} & G_{12} & G_{22} & G_{26} & F_{12} & F_{22} & F_{26} & H_{12} & H_{22} & H_{26} \\
E_{16} & E_{26} & E_{66} & G_{16} & G_{26} & G_{66} & F_{16} & F_{26} & F_{66} & H_{16} & H_{26} & H_{66}
\end{bmatrix}
\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \varepsilon_x^{o*} \\ \varepsilon_y^{o*} \\ \gamma_{xy}^{o*} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \kappa_x^* \\ \kappa_y^* \\ \kappa_{xy}^* \end{bmatrix}
\tag{10}
$$

and,

$$
\begin{bmatrix} Q_x \\ Q_y \\ S_x \\ S_y \\ Q_x^* \\ Q_y^* \end{bmatrix}
=
\begin{bmatrix}
A_{55} & A_{45} & B_{55} & B_{45} & D_{55} & D_{45} \\
A_{45} & A_{44} & B_{45} & B_{44} & D_{45} & D_{44} \\
B_{55} & B_{45} & D_{55} & D_{45} & E_{55} & E_{45} \\
B_{45} & B_{44} & D_{45} & D_{44} & E_{45} & E_{44} \\
D_{55} & D_{45} & E_{55} & E_{45} & F_{55} & F_{45} \\
D_{45} & D_{44} & E_{45} & E_{44} & F_{45} & F_{44}
\end{bmatrix}
\begin{bmatrix} \varphi_x \\ \varphi_y \\ \gamma_{xz}^{o*} \\ \gamma_{yz}^{o*} \\ \varphi_x^* \\ \varphi_y^* \end{bmatrix}
\tag{11}
$$

All coefficients in ***A***, ***B***, ***D***, ***E***, ***F***, ***G***, and ***H*** groups are defined as follows:

$$A_{ij} = \sum_{L=1}^{NL} Q_{ij}(h_L - h_{L-1}) \qquad i, j = 1, 2, 6 \text{ or } i, j = 4, 5 \tag{12 a}$$

$$B_{ij} = (1/2)\sum_{L=1}^{NL} Q_{ij}(h_L^2 - h_{L-1}^2) \qquad i, j = 1, 2, 6 \text{ or } i, j = 4, 5 \tag{12 b}$$

$$D_{ij} = (1/3)\sum_{L=1}^{NL} Q_{ij}(h_L^3 - h_{L-1}^3) \qquad i, j = 1, 2, 6 \text{ or } i, j = 4, 5 \tag{12 c}$$

$$E_{ij} = (1/4)\sum_{L=1}^{NL} Q_{ij}(h_L^4 - h_{L-1}^4) \qquad i, j = 1, 2, 6 \text{ or } i, j = 4, 5 \tag{12 d}$$

$$F_{ij} = (1/5)\sum_{L=1}^{NL} Q_{ij}(h_L^5 - h_{L-1}^5) \qquad i, j = 1, 2, 6 \text{ or } i, j = 4, 5 \tag{12 e}$$

$$G_{ij} = (1/6)\sum_{L=1}^{NL} Q_{ij}(h_L^6 - h_{L-1}^6) \qquad i, j = 1, 2, 6 \tag{12 f}$$

$$H_{ij} = (1/7)\sum_{L=1}^{NL} Q_{ij}(h_L^7 - h_{L-1}^7) \qquad i, j = 1, 2, 6 \tag{12 g}$$

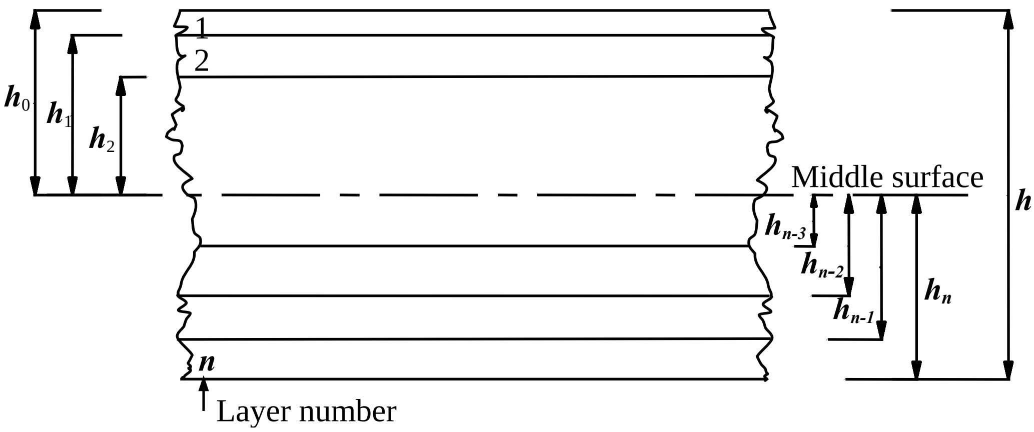

Figure (3): Geometry of an NL-layered laminate [Jones,1999]()

DYNAMIC EQUILIBRIUM EQUATION

The dynamic equilibrium Equations are obtained by using the principle of virtual work which states that for any arbitrary kinematically consistent set of displacements, the internal virtual work done by stresses through virtual strains must be equal to that done by the external forces irrespective of the material behavior as [**Cook**, **1995**](8):

$$\int_v (d\varepsilon)^T \sigma\, dv = \int_{s_t} (du)^T P_t\, ds + \int_v (du)^T (P_b - \rho\ddot{u} - C\dot{u})dv \tag{13}$$

where du is a vector of virtual displacements, $d\varepsilon$ is the vector of associated virtual strains and σ is the vector of actual stresses. The term P_t is a vector of surface tractions acting on the portion s_t of the boundary S. Vectors $P_b, \rho\,\ddot{u}$ and $C\,\dot{u}$ are the body, inertial and damping forces respectively. The symbol ($\dot{}$) denotes differentiation with respect to time. ρ is the mass density and C is the damping parameter.

For the finite element representation, the displacements, velocities and accelerations $u, \dot{u}$ and $\ddot{u}$ can be defined in terms of the nodal variables $d, \dot{d}$ and $\ddot{d}$ by the expressions:

$$u = \sum_{i=1}^{m} N_i(\xi,\eta)\, d_i = Nd \quad , \; du = N\,\delta d \tag{14}$$

$$\dot{u} = \sum_{i=1}^{m} N_i(\xi,\eta)\, \dot{d}_i = N\,\dot{d} \tag{15}$$

$$\ddot{u} = \sum_{i=1}^{m} N_i(\xi,\eta)\, \ddot{d}_i = N\,\ddot{d} \tag{16}$$

where $\boldsymbol{u}=\sum_{i=1}^{m} N_i(\xi,\eta)\, d_i = Nd$, N_i is the shape functions for $\boldsymbol{i}$ node, and $\boldsymbol{m}$ is the number of nodes.

With standard strain-nodal displacement matrix $[B]$, the virtual strain vector can be related to the nodal displacements as:

$$d\varepsilon=\sum_{i=1}^{m}[B]_i\,\delta d_i=[B]\delta d \tag{17}$$

Upon substitution of Equations (14-17) into Equation (13) then:

$$\delta d^T\left[[M]\ddot{d}+[C]\dot{d}+[K]d\right]=\delta d^T\{f_e(t)\} \tag{18}$$

in which the mass matrix $[M]$, the damping matrix $[C]$, the stiffness matrix $[K]$ and the external applied vector $\{f_e(t)\}$ have the following element contributions:

$$[M_e]=\int_{Ve} N^T\rho\, N\, dV \tag{19}$$

$$[C_e]=\int_{Ve} N^T C\, N\, dV \tag{20}$$

$$[K_e]=\int_{Ve}[B]^T[D][B]dV \tag{21}$$

$$f_e(t)=\int_{se} N^T P_t\, ds+\int_{Ve} NP_b\, dV \tag{22}$$

where s_e and V_e denote the surface and volume of the element under consideration. As δd^T is arbitrary, then Equation (18) may be written as:

$$[M]\{\ddot{d}\}+[C]\{\dot{d}\}+[K]\{d\}=\{f_e(t)\} \tag{23}$$

Equation (23) is the dynamic equilibrium Equation for a single or multi-degree of freedom system.

FORMULATION OF ELEMENT MASS MATRIX

The kinetic energy of the element (e) can be expressed as follows:

$$TI^e=\frac{1}{2}\int_A\{\dot{d}\}^T[m]\{\dot{d}\}dA \tag{24}$$

The velocity vector within an element is discretized such that:

$$\{\dot{d}\}=\sum_{i=1}^{NN}N_i\{\dot{d}\}, \text{ NN: number of nodes.} \tag{25}$$

By substituting Equation (25) into Equation (24), one gets:

$$TI^e=\frac{1}{2}\sum_{i=1}^{NN}\{\dot{d}\}^T\int_A N_i^T[m]N_i\; dA\{\dot{d}\} \tag{26}$$

Thus,

$$[M]^e=\int_A[N]^T[m][N]dA=\int_{-1}^{1}\int_{-1}^{1}[N]^T[M][N]|J|\,d\xi\, d\eta \tag{27}$$

The mass matrix for nine degrees of freedom per node is:

$$[\boldsymbol{m}]_{9\times 9} = \begin{bmatrix} \boldsymbol{I}_1 & & & & & & & & 0 \\ & \boldsymbol{I}_1 & & & & & & & \\ & & \boldsymbol{I}_1 & & & & & & \\ & & & \boldsymbol{I}_2 & & & & & \\ & & & & \boldsymbol{I}_2 & & & & \\ & & & & & \boldsymbol{I}_3 & & & \\ & & & & & & \boldsymbol{I}_3 & & \\ & & & & & & & \boldsymbol{I}_4 & \\ 0 & & & & & & & & \boldsymbol{I}_4 \end{bmatrix} \tag{28}$$

For layered plates, the element mass matrix can be written as follows:

$$[\boldsymbol{M}] = \sum_{L=1}^{NL} [\boldsymbol{M}]^{e} \tag{29}$$

where in the above Equation (28), $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$, and $\boldsymbol{I}_4$ are translation inertia, rotary inertia, and respectively higher order inertia terms, and these are given by:

$$(\boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3, \boldsymbol{I}_4) = \sum_{L=1}^{NL} \int_{h_{L-1}}^{h_L} (1, z^2, z^4, z^6)\, \rho^{L}\, dz \tag{30}$$

where ρ^{L} is material density of L-th layer.

FORMULATION OF DAMPING PROPERTIES

The most common form for the representation of the damping matrix [$\boldsymbol{C}$] is the so-called Rayleigh-type damping(3) which was given as;

$$[\boldsymbol{C}] = a_o [\boldsymbol{M}] + a_1 [\boldsymbol{K}] \tag{31}$$

in which (a_o and a_1) are arbitrary proportionality factors, which make the damping matrix satisfy the orthogonality condition with respect to the modal matrix [$\boldsymbol{\Phi}$] in the same way of the orthogonality conditions for the mass and stiffness matrices that is(5):

$$\begin{aligned} \{\Phi\}^{T} [\boldsymbol{M}] \{\Phi\} &= [\boldsymbol{I}] \\ \{\Phi\}^{T} [\boldsymbol{K}] \{\Phi\} &= [\Lambda] \\ \{\Phi\}^{T} [\boldsymbol{C}] \{\Phi\} &= 2[\gamma][\Lambda]^{1/2} \end{aligned} \tag{32}$$

where

$\{\Phi\}$ **:** The modal matrix whose columns represent the natural modal shapes and the superscript ($\boldsymbol{T}$) denotes transpose.

$[\boldsymbol{I}]$ **:** Identity matrix.

$[\Lambda]$ **:** Spectral matrix, which is a diagonal matrix with elements representing the squares of the natural frequencies (ω_i^2).

$[\gamma]$ **:** Modal damping matrix which is also a diagonal matrix with elements representing the damping ratios for the system modes (γ_i)

Premultiplying Equation (37) by $\{\Phi\}^T$ and postmultiplying it by $\{\Phi\}$ yields:

$$\{\Phi\}^T[C]\{\Phi\} = a_o\{\Phi\}^T[M]\{\Phi\} + a_1\{\Phi\}^T[K]\{\Phi\} \tag{33}$$

Substituting Equations (38) into Equation (39) gives;

$$2[\gamma][\Lambda]^{1/2} = a_o[I] + a_1[\Lambda] \tag{34}$$

The two factors, a_o and a_1 can be determined by specifying the damping ratios for two modes for example 1 and 2, and substituting into Equation (34) as[12]:

$$2\gamma_1\omega_1 = a_o + a_1\omega_1^2 \tag{35}$$

$$2\gamma_2\omega_2 = a_o + a_1\omega_2^2 \tag{36}$$

where ω_1 and ω_2 are the natural frequencies for modes 1 and 2 respectively. By solving the above two Equations one can get:

$$a_o = \frac{2\omega_1\omega_2(\omega_2\gamma_1 - \omega_1\gamma_2)}{(\omega_2^2 - \omega_1^2)} \tag{37}$$

$$a_1 = \frac{2(\omega_2\gamma_2 - \omega_1\gamma_1)}{(\omega_2^2 - \omega_1^2)} \tag{38}$$

Then, the values of a_o and a_1 are substituted into Equation (31) to get the required damping matrix.

FORCED VIBRATION ANALYSIS

The calculation of the nonlinear dynamic response of structure of structures including instability or buckling phenomena has received considerable attention and a good amount of literature has appeared on this subject. The nonlinear dynamic analysis depends largely on solving the following Equations:

$$[M]\{\ddot{d}(t)\} + [C]\{\dot{d}(t)\} + [K_T]\{d(t)\} = \{F(t)\} \tag{39}$$

in which $[K_T]$ is the tangent stiffness matrix of the plate (or structure) and depends on the current displacements and stresses. The most conventional implicit time integration procedures is **Newmark** method. After solving Equation (39) at time (t+Δt) for displacements, velocities, and accelerations, the following equation as:

$$\begin{aligned}([K_T] + a_o[M] + a_1[C])\{d\}_{t+\Delta t} = \{F(t)\}_{t+\Delta t} &+ [M]\left(a_2\{\dot{d}\}_t + a_3\{\ddot{d}\}_t\right) \\ &+ [C]\left(a_4\{\dot{d}\}_t + a_5\{\ddot{d}\}_t\right)\end{aligned} \tag{40}$$

For convenience, the following is used:

$$[K_T]_{eff} = [K_T] + a_o[M] + a_1[C] \tag{41}$$

and,

$$\{F(t)\}_{eff} = \{F(t)\}_{t+\Delta t} + [M]\left(a_2\{\dot{d}\}_t + a_3\{\ddot{d}\}_t\right) + [C]\left(a_4\{\dot{d}\}_t + a_5\{\ddot{d}\}_t\right) \tag{42}$$

So, Equation (42) may be written in the form:

$$\left[\boldsymbol{K_T}\right]_{\boldsymbol{eff}}\{\boldsymbol{d}\}_{\boldsymbol{t+\Delta t}} = \{\boldsymbol{F(t)}\}_{\boldsymbol{eff}} \tag{43}$$

For a linear system, $\left[\boldsymbol{K_T}\right]_{\boldsymbol{eff}}$ will be constant during the analysis at any time, while in the nonlinear analysis, $\left[\boldsymbol{K_T}\right]_{\boldsymbol{eff}}$ is a function of current displacement vector $\{\boldsymbol{d}\}$. Therefore, an iterative procedure must be used to define $\left[\boldsymbol{K_T}\right]_{\boldsymbol{eff}}$. In the nonlinear analysis, it is more useful to put Equation (43) in increment form. For such purpose, Equation (43) may be rewritten as:

$$\left[\boldsymbol{\hat{K}_T}\right]\{\Delta \boldsymbol{d}\} = \left\{\Delta \boldsymbol{\hat{F}(t)}\right\} \tag{44}$$

in which $\left[\boldsymbol{\hat{K}_T}\right]$ is the effective stiffness matrix and $\left\{\Delta \boldsymbol{\hat{F}(t)}\right\}$ is the effective load vector. Equation (44) is solved by an iterative procedure like Equation (40). It may be noted that Equation (40) may be used for solving linear problems, while for nonlinear problems, Equation (44) should be used.

Solving Equation (44) for $\{\Delta \boldsymbol{d}\}$, approximate values for accelerations, velocities and displacements may be given as:

$$\begin{aligned}
\{\ddot{\boldsymbol{d}}\}_{\boldsymbol{t+\Delta t}} &= \boldsymbol{a_o}\{\Delta \boldsymbol{d}\} - \boldsymbol{a}_2\{\dot{\boldsymbol{d}}\}_{\boldsymbol{t}} - \boldsymbol{a}_3\{\ddot{\boldsymbol{d}}\}_{\boldsymbol{t}} \\
\{\dot{\boldsymbol{d}}\}_{\boldsymbol{t+\Delta t}} &= \boldsymbol{a}_1\{\Delta \boldsymbol{d}\} - \boldsymbol{a}_4\{\dot{\boldsymbol{d}}\}_{\boldsymbol{t}} - \boldsymbol{a}_5\{\ddot{\boldsymbol{d}}\}_{\boldsymbol{t}} \\
\{\boldsymbol{d}\}_{\boldsymbol{t+\Delta t}} &= \{\boldsymbol{d}\}_{\boldsymbol{t}} + \{\Delta \boldsymbol{d}\} \ ;
\end{aligned} \tag{45}$$

where

$$\boldsymbol{a_o} = \frac{1}{\beta(\Delta \boldsymbol{t})^2},\ \boldsymbol{a}_1 = \frac{\gamma}{\beta(\Delta \boldsymbol{t})},\ \boldsymbol{a}_2 = \frac{1}{\beta(\Delta \boldsymbol{t})},\ \boldsymbol{a}_3 = \frac{1}{2\beta} - 1$$

$$\boldsymbol{a}_4 = \frac{\gamma}{\beta} - 1,\ \boldsymbol{a}_5 = \frac{\Delta \boldsymbol{t}}{2}\left(\frac{\gamma}{\beta} - 2\right)$$

APPLICATIONS AND DISCUSSIONS

Several plates are analyzed to study the different effects on the large displacement dynamic behavior of plates with some comparison with other researchers.

Comparison with available theoretical investigation of composite plate

Clamped supported square angle-ply laminated plate under transverse suddenly applied constant dynamic loading

A square angle-ply ($0^o/45^o/90^o$/core/$90^o/45^o/30^o/0^o$) sandwich laminated plate with clamped edges and subjected to a suddenly applied uniformly transverse load was analyzed and compared with **Kommineni** and **Kant** [**1993**][0]. The following layer material properties are used in the analysis: for face sheets (Graphite/epoxy prereg system) ($\boldsymbol{E}_1$=130.8 GPa; $\boldsymbol{E}_2$=10.6 GPa $\boldsymbol{G}_{12}$=$\boldsymbol{G}_{13}$=6 GPa; $\boldsymbol{G}_{23}$=3.9 GPa; $\boldsymbol{\nu}_{12}$=0.28; and ρ=15.8 kN.sec^2/m^4); for core sheet (US Commercial al. honeycomb, ¼ in cell size, 0.003 in foil) ($\boldsymbol{G}_{13}$=0.5206 GPa; $\boldsymbol{G}_{23}$=0.1772 GPa; ρ=1.009 kN.sec^2/m^4). The time step is (Δ***t***=0.000025 sec),and applied load (***q***=50 kN/m^2). The geometry properties are (***a***=1.0 m, ***a***/***b***=1, and ***h***=0.01m, at top three stiff layers, thickness of each layer=0.025 ***h***, at bottom four stiff layer, thickness of each layer=0.08125 ***h***, and thickness of core=0.6 ***h***). **Kommineni** and **Kant** used nine-node isoparametric Lagrangian elements with nine-node degrees of freedom per node and divided the full plate into (4×4) element mesh.

In the present study, the full laminated plate is modeled by (4×4) element mesh with nine-node isoparametric Lagrangian element and nine degrees of freedom per node. A consistent mass matrix and **Newmark** integration method with α=1/2, and β=1/4 were used in the present study.

Figure (4) shows the time history curve for the clamped angle-ply laminated plate under transverse suddenly applied load. From this figure, it can be noticed that good agreement with other study exists with a difference not exceeding (1%).

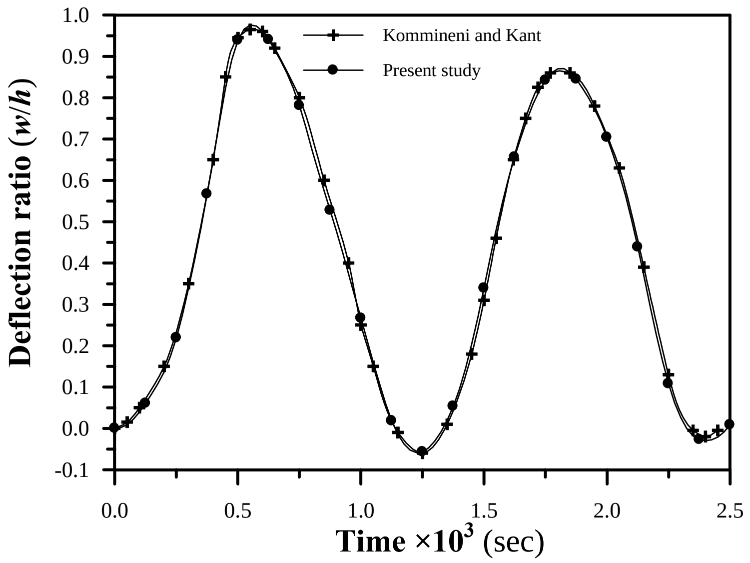

Figure (4): Central deflection ratio-time curve of a clamped square sandwich composite plate under transverse constant dynamic loading,(b/h=100, Δt=0.000025sec, q=50 kN/m^2)

A simply supported square plate with slenderness ratio (***b***/***h***=100), and with symmetric cross-ply and antisymmetric cross-ply arrangements, were chosen to study the effect of number of layers on the large displacement dynamic behavior of a laminated composite plate under in-plane constant dynamic loading. The initial imperfection is ($\boldsymbol{w_o/h}$= 0.1) by which the shape is considered to be a sinusoidal curve.

Figures (5) and (6) present the time history curve and show that for the same volume of the plate, the response (deflection) will decrease about (15%) for the symmetric cross-ply and about (29%) for the antisymmetric cross-ply plates where with increasing the number of layers (3-10) for the symmetric cross-ply and (2-10) for the antisymmetric cross-ply arrangements, the stiffness increase may be related to the increase of the number of the reinforced layers. Thus, extension and bending stiffness will increase; and therefore, the amplitude will decrease. Also, the increase of the number of layers will give a better distribution of orthogonal stiffness through the depth. From these figures, it can be seen that the increase of the number of layers more than (8 layers) for the symmetric cross-ply and the antisymmetric cross-ply plates have slight effect on increasing the stiffness of the plate.

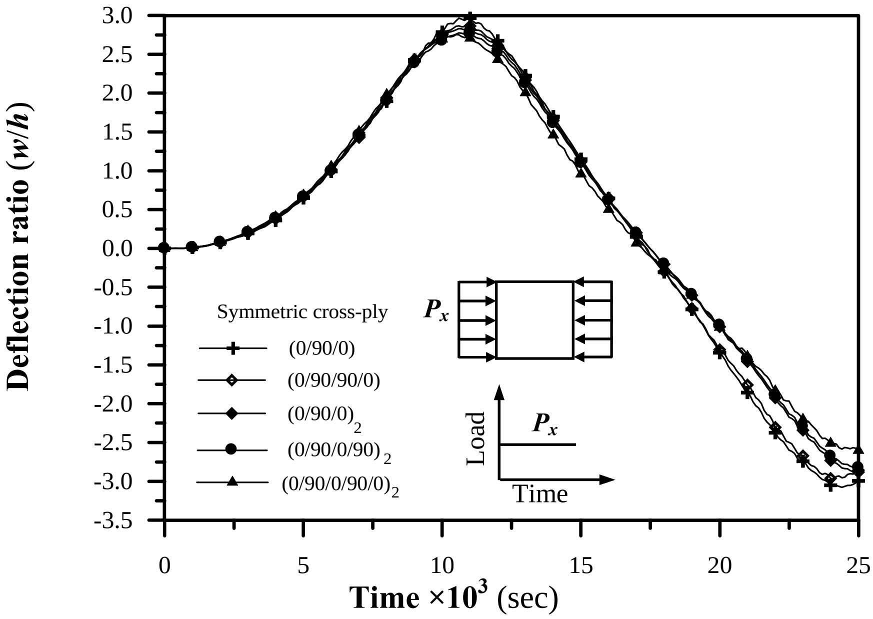

Figure (5): Effect of number of layers on the large displacement analysis of symmetric cross-ply laminated plate under in-plane constant dynamic loading ratio (P_x/P_u=0.4), (b/h=100, t=0.0001, w_o/h=0.1)

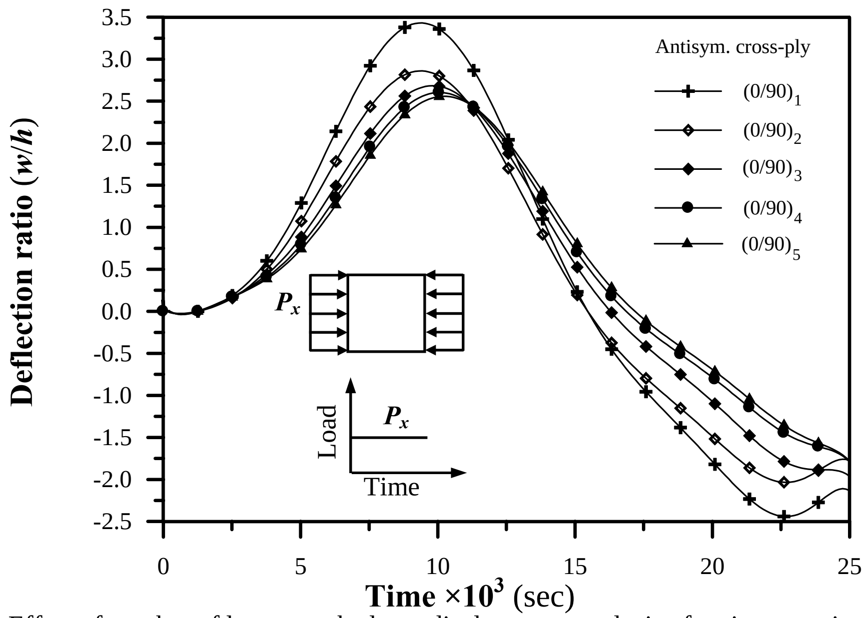

Figure (6): Effect of number of layers on the large displacement analysis of antisymmetric cross-ply laminated plate under in-plane constant dynamic loading ratio (P_x/P_u=0.4), (b/h=100, t=0.0001, w_o/h=0.1)

To study the effect of shear deformation on the large displacement dynamic analysis of a laminated composite plate under in-plane constant dynamic loading, a simply supported square plate with slenderness ratio (***b***/***h***=20), and with symmetric cross-ply antisymmetric cross-ply arrangements and with eight layers was analyzed. The initial imperfection is ($\boldsymbol{w_o/h}$= 0.1) by which the shape is considered to be a sinusoidal curve.

Figures (7) and (8) present the time history curves for the symmetric cross-ply, and for the antisymmetric cross-ply laminated composite plates by taking the through-thickness shear deformation through the degrees of freedom of the element. From these figures, it can be noticed that increasing the number of degrees of freedom per node from five degrees to nine degrees will increase the central deflection about (16%) for symmetric cross-ply and about (20%) for antisymmetric cross-ply plates.

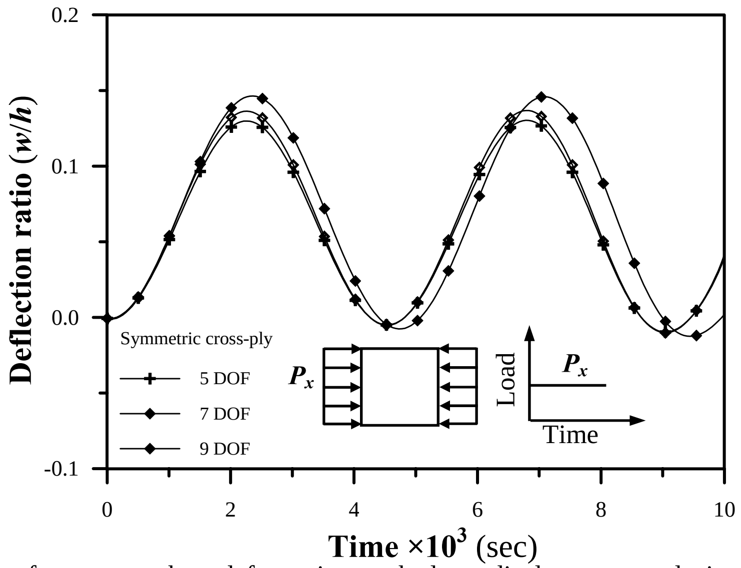

Figure (7): Effect of transverse shear deformation on the large displacement analysis of symmetric cross-ply laminated plate under in-plane constant dynamic loading ratio (P_x/P_u=0.3), (b/h=20, t=0.0001, w_o/h=0.1,P_u=18563 kN/m)

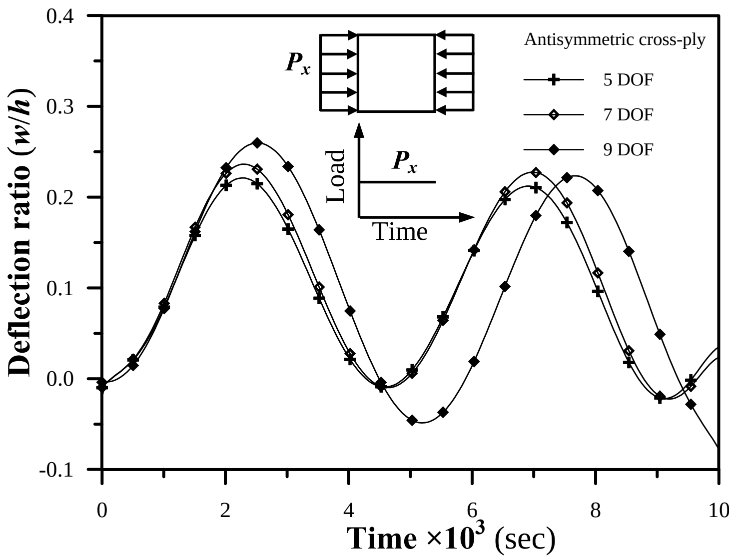

Figure (8): Effect of transverse shear deformation on the large displacement analysis of antisymmetric cross-ply laminated plate under in-plane constant dynamic loading ratio (P_x/P_u=0.3), (b/h=20, t=0.0001, w_o/h=0.1,P_u=16347 kN/m)

To study the effect of damping on the large displacement elastic-plastic dynamic behavior of composite plates, two examples are considered. The first one is a simply supported square plate with symmetric cross-ply lamination with eight layers and under in-plane dynamic loading. The second one is a simply supported square plate with antisymmetric cross-ply lamination with eight layers and under in-plane dynamic loading. Different values of damping factor (0.05-0.1) are considered in the present study. The initial imperfection shape is considered to be a sinusoidal curve. The following geometry and layer material properties of high graphite epoxy are used in the analysis: ($\boldsymbol{E}_1$=172.5 GPa***;*** $\boldsymbol{E}_2$=7.08 GPa; $\boldsymbol{G}_{12}$=$\boldsymbol{G}_{13}$=3.45 GPa, $\boldsymbol{G}_{23}$=1.38 GPa; ρ=15.8 kN.sec^2/m^4)(). The geometry properties are (***a***=1.0 m, ***a***/***b***=1).

Figure (9) and (10) present the time history curve for a simply supported square plate with symmetric and antisymmetric cross-ply lamination under in-plane constant loading. It is noticed that the response (deflection) decreases with the increase of the damping factor. Also, the plate shows no oscillation about the static deflection position, this means that the plate is under the critical damping ratio

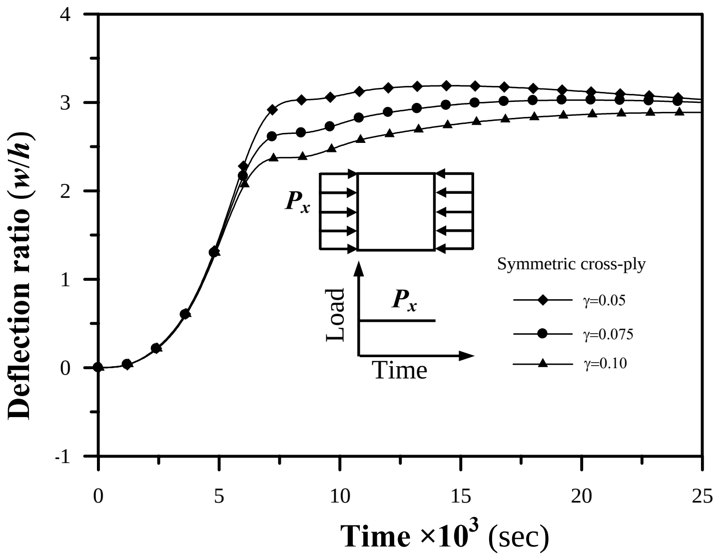

Figure (9):Effect of damping factor on the large displacement analysis of a simply supported square symmetric cross-ply plate under in-plane constant dynamic loading, (b/h=100, t=0.0001, w_o/h=0.1, P_x/P_u=0.65, P_u=972.4 kN/m)

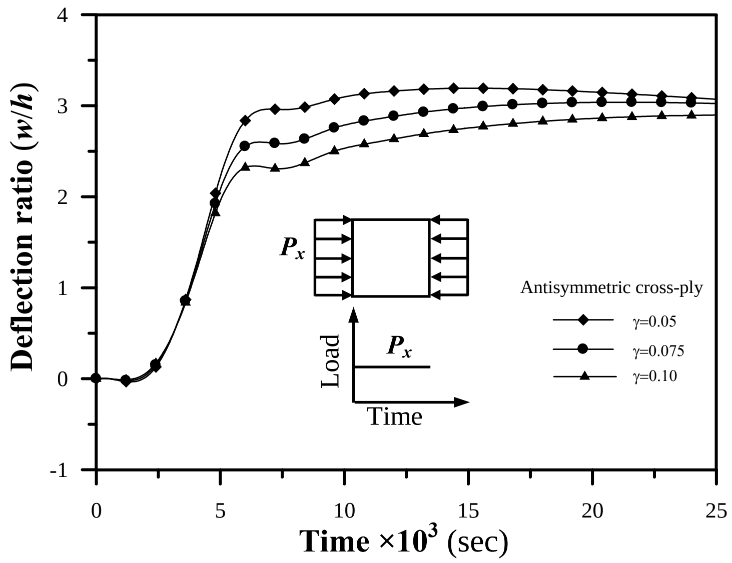

Figure (10):Effect of damping factor on the large displacement analysis of a simply supported square antisymmetric cross-ply plate under in-plane constant dynamic loading, (b/h=100, Δt=0.0001, wo/h=0.1, Px/Pu=0.65, Pu=960 kN/m)

CONCLUSIONS

A nonlinear finite element method is adopted for the large displacement dynamic analysis of anisotropic plates under in-plane compressive load. Damping property is considered by using Rayleigh type damping which is linearly related to the mass and the stiffness matrices. Newmark integration method is used for solving the dynamic equilibrium equations. The effects of initial imperfection, orthotropy of individual layers, fiber's orientation angle, type of loading, damping factor, and on the large displacement dynamic analysis are considered. The conclusion it is shown that the antisymmetric cross-ply laminated plate has a damping rate faster than the symmetric cross-ply laminated plate and if damping is considered and if the response of the plate shows no oscillation about the static deflection position, it means that the damping factor is below the critical damping factor. So, noticed that the central deflection increasing with increasing the degree of freedom per node.

REFERENCES

[1] Ali, N. H., "Finite Element Dynamic Analysis of Laminated Composite Plates Including Damping Effect", *M.Sc. Thesis*, University of Babylon, Hilla, Iraq, 2004.

[2] Akay, H. "Dynamic Large Deformation Analysis of Plates Using Mixed Finite Elements" *Comp. & Struct.*, Vol.11, 1980, pp1-11.

[3] Ammash, H. K., "Nonlinear Static and Dynamic Analysis of Laminated Plates Under In-plane Forces", *Ph.D. Thesis*, University of Babylon, Hilla, Iraq, 2008.

[4] Azevedo, R.L. and Awruch, A.M. "Geometric Nonlinear Dynamic Analysis of Plates and Shells Using Eight-Node Hexahedral Finite Element with Reduced Integration", *J. Braz. Soc. Mech. Sci.*, Vol.21, No.3, 1999, pp.1-22.

[5] Bathe, K.J., and Ozdemir, H. "Elastic-Plastic Large Deformation Static and Dynamic Analysis.", *Comp. & Struct.*, Vol.6, No.2, 1975, pp81-92.

[6] Jones, R.M., "Mechanics of Composite Materials", *Second Edition*, Taylor and Francis Inc., U.S.A., 1999.

[7] Kao, R., "Nonlinear Dyanmic Buckling of Spherical Caps with Initial Imperfections", *Comp. & Struct.*, Vol.12, 1980, pp49-63.

[8] Kaw, A., "Mechanics of Composite Materials", *Second Edition*, Taylor and Francis Group, LLC, 2006.

[9] Khante, S. N., Rode, V., and Kant, T., "Nonlinear Transient Dynamic Response of Damping Plates Using a Higher Order Shear Deformation Theory", *Nonlinear Dynamics*, Vol.47, 2007, pp38-403.

[10] Kommineni, J. R., and Kant, T. "Geometrically Non-linear Transient Co Finite Element Analysis of Composite and Sandwich Plates with a Refined Theory." *Struct. Eng. And Mech.*, Vol.1, No.1, 1993, pp87-102.

[11] Pica, A., Wood, R.D., and Hinton, E. "Finite Element Analysis of Geometrically Nonlinear Plate Behavior Using a Mindlin Formulation." *Comp. & Struct.*, Vol.11, 1979, pp.203-215.

[12] Pytet, M., "Introduction to Finite Element Vibration Analysis", 1990.

[13] Tao, Z., Tu-guang, L., Yao,Z., and Jio-zhi,L. "Nonlinear Dynamic Buckling of Stiffened Plates under In-plane Impact Load.", *J. Zhejiang University Science*, Vol.5, No.5, 2004, pp609-617.

[14] Weller, T., Abramovich, H., and Yaffe, R., "Dynamic Buckling of Beams and Plates Subjected to Axial Impact", *Comp. & Struct.*, Vol.32, No.3/4, 1989, pp835-851.